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Advanced Computational Methods for Solving Multi-Scale Nonlinear Engineering Problems

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ABSTRACT

The classical integer-order transport equations, like the advection-diffusion equation, are not always useful in representing the anomalous transport process in complex environmental systems, especially in non-Fickian non-homogeneous porous media where long-range dependence and memory effects are predominant. To overcome these pitfalls, this paper introduces a fractional-order differentiation modeling system in order to implement non-local temporal and spatial dynamics. This is achieved by developing a generalized form of the transport model based on Caputo time-fractional and Riesz space-fractional derivatives in order to describe sub-diffusive and super-diffusive processes. It is the proposed model, which is numerically solved by a Grünwald-Letnikov-based discretization scheme (with a stable finite difference approach). The results of the simulations show that the fractional model is much better in predicting the concentration profiles than the classical models, and is effective in capturing the heavy tails and late transport behavior. The parametric analysis shows how the dynamics of a system are affected by the choice of the fractional orders, which can be used to understand the mechanisms of transport more thoroughly. The suggested framework provides a strong and physically uniform device to model anomalous transport used in the environmental engineering contexts such as groundwater contamination and pollutant dispersion.

1. INTRODUCTION

Environmental transport phenomena are important in a broad variety of engineering applications including groundwater contamination, pollutant dispersion, soil remediation and atmospheric transport processes. Proper modeling of such systems is needed to predict the destiny of contaminants and their movement, environmental risk, and mitigation measures. Historically, these have been modeled by classical integer-order equations, especially the advection diffusion equation (ADE) that is formulated on the basis of Fickian diffusion laws and homogeneous media and local interactions [5], [6].

But experimental and field observations have shown that in actual environmental systems transport behavior is frequently very non-classical. Fractured geological formations, high heterogeneity of porous media, and other complex subsurface environments have anomalous transport properties of contaminants including non-Gaussian plume spreading, heavy-tailed concentration distributions and long memory effects [9], [12]. Often called non-Fickian or anomalous transport cannot be effectively modeled by standard integer-order approaches, resulting in poor predictions and little physical realism.

Fractional calculus has arisen to counter these shortcomings and has become a useful mathematical tool to describe systems with memory and non-local spatial interactions [1]–[3], [11]. Fractional-order differential equations are extensions of classical models, and make use of non-integer-order derivatives, which allow models of sub-diffusive and super-diffusive processes to be represented. Fractional models are a more loose and physically coherent description of the process of anomalous transport, especially in complex and multi-scale systems, when the classical assumptions fail [10].

Although there has been immense advancement in using fractional calculus to transport phenomena, there are still a number of challenges. Numerous available studies rely more on theoretical formulations without the thorough validation in realistic environmental conditions [4], [8]. Also, effective and robust numerical techniques to solve fractional differential equations on a large scale in engineering applications are actively being developed [5], [10].

Systematic study of the effect of fractional parameters on transport behavior is also required to gain a more physical intuition as to their importance.

This paper presents a fractional-order differential modelling framework to explore the dynamics of anomalous transport in environmental engineering systems. The model proposed uses time-fractional and space-fractional derivatives of Caputo and Riesz, respectively, to include both the effects of temporal memory and spatial heterogeneity. To achieve numerical stability and computational efficiency, a powerful numerical solution method that involves the use of Grunwald-Letnikov discretization is applied. Simulation studies and comparison with classical transport models prove this model correct [6], [7]. The major advancements of the work are: (i) a generalized fractional transport model was developed, (ii) a stable numerical solution scheme was developed, (iii) the influence of fractional parameters on the transport dynamics and (iv) the application of the results to environmental problems such as pollutant transport in porous media.

2. LITERATURE REVIEW

The advection-diffusion equation (ADE) of environmental engineering has largely been a classical basis of the study of mass transport, being based upon the assumption of Fickian diffusion and used of integer-order derivatives to quantify transport processes [5], [6]. Since it is mathematically simple, and can be analyzed analytically, the ADE has found widespread use in modeling the contaminant movement in groundwater, the dispersal of pollutants in the atmosphere, and the transport of solutes in soils. Nevertheless, its basic premise of local balance and homogeneous media restricts its use in complex environmental systems. On heterogeneous and fractured media, the ADE usually does not accurately simulate the dynamics of the plume, which causes differences between model forecasts and observations. To overcome these shortcomings, a lot of studies have been done on anomalous transport behavior that does not conform to classical Fickian diffusion [9], [12]. Anomalous transport is defined as non-linear mean square displacement, heavy-tailed probability distributions and long-range temporal and spatial correlations. Sub- diffusive processes a slower rate of transport than predicted by classical models are usually found in highly heterogeneous porous media, whereas super- diffusive behaviour is observed in systems having preferential routes to flow or a turbulent mix. These effects underline the necessity of state-of-the-art modeling methodologies capable of modeling the effects of memory and non-local interactions that are inherent in environmental systems.

The application of fractional calculus has become important in modeling the anomalous transport processes because of its capability to introduce the effects of memory and spatial heterogeneity by introducing non-integer derivatives [1] -[3], [11]. Time-fractional and space-fractional advection-diffusion equations are examples of fractional-order models that have been effectively used in different environmental fields including groundwater hydrology, transport of contaminants in porous media and modeling of atmospheric dispersion [6], [9]. The temporal memory effects can be described using Caputo and RiemannLiouville derivatives while long-range spatial interactions are described using Riesz fractional derivatives. These models have been shown to be accurate in non-Fickian behaviour in transport than their classical counterparts [10].

Although this has been achieved, a number of challenges exist in the practical implementation of fractional-order models in environmental engineering. Most of the literature is restricted to theoretical investigation or simplified conditions, and lacks verification with realistic environmental conditions [4], [8]. Also, the computational complexity involved in the calculation of the derivatives of the fractional is a problem to large-scale simulations, especially multi-dimensional systems [5], [10]. It is also deficient of the thorough studies that can analytically examine the role played by fractional parameters on the transport dynamics as well as physical interpretation of such behavior of the real world system. Thus, it is evident that a strong and tested fractional-order modeling framework is required to incorporate a correct mathematical formulation, a good numerical implementation and useful application to environmental transport issues. This paper will fill these gaps by making a complete model of the fractional differential, with solid support of stable numerical solution method, and elaborate explanation of aberrant transport in environmental engineering systems.

3. Mathematical Modeling and Problem Formulation

3.1 Classical Transport Model

The classical advection diffusion equation (ADE) is often used to describe transport phenomena in environmental engineering to describe the joint effect of advective transport and diffusive spreading. This model also supposes that the movement of the contaminants is caused by bulk fluid movement, and concentration gradient, which is in line with the principles of Fickian diffusion. Using the homogeneous medium and local interaction, the one dimensional ADE is given as:

$\frac{\partial C(x,t)}{\partial t} + v\frac{\partial C(x,t)}{\partial x} = D\frac{\partial^2 C(x,t)}{\partial x^2}$,_______________(1)

In Eq. (1), $C(x,t)$ denotes the concentration of the transported substance as a function of spatial coordinate $x$and time t , v the average advective velocity, and D is the diffusion coefficient associated with molecular diffusion and mechanical dispersion.

The term $\frac{\partial C(x,t)}{\partial t}$ represents the temporal variation of concentration, while the advective term $v\frac{\partial C(x,t)}{\partial x}$describes transport due to bulk fluid motion. The diffusive $D\frac{\partial^2 C(x,t)}{\partial x^2}$accounts for spreading driven by concentration gradients.Despite the simplicity and analytical convenience of the Equ. (1), it is based on more limiting assumptions like spatial homogeneity, local equilibrium, and the behavior of Gaussian dispersion. These assumptions restrict its ability to apply to real environmental systems, in which heterogeneous structures and complicated interactions often can give rise to anomalous transport behavior. As a result, the classical ADE does not bring into focus the heavy-tailed concentration distributions and memory-dependent dynamics, leading to the creation of models of fractional order.

3.2 Fractional-Order Model

In order to address the shortcomings of the classical advection diffusion equation to model the anomalous transport behavior, a generalized form of the fractional-order model is proposed. Non-integer order derivatives allow a memory effect and long-range spatial interactions to be incorporated using fractional calculus. The fractional-order advection-diffusion equation could be written as:

$CD_t^{\alpha} C(x,t) + v\frac{\partial C(x,t)}{\partial x} = D\frac{\partial^{\beta} C(x,t)}{\partial x^{\beta}}$,______(2)

In Eq. (2), CD_t^{α} denotes the Caputo fractional derivative of order$\alpha \in [0,1]$, which accounts for temporal memory effects in the transport process. The term $\frac{\partial^{\beta}}{\partial x^{\beta}}$ represents the Riesz fractional derivative of order$\beta \in [0,1]$, capturing non-local spatial interactions and long-range particle movement.The parameter αgoverns the degree of memory in the system. When$\alpha = 1$, the model reduces to classical time behavior, whereas $\alpha < 1$corresponds to sub-diffusive transport characterized by delayed spreading and retention effects. Similarly, the parameter β\betaβ controls the spatial diffusion behavior. For$\beta = 2$, the classical diffusion term is recovered, while $\beta < 2$ introduces anomalous spatial transport, often associated with heavy-tailed distributions and super-diffusive effects.

The fractional time derivative $CD_t^{\alpha} C(x,t)$ in Eq. (2) reflects the influence of past states on current transport dynamics, thereby modeling memory-dependent processes commonly observed in heterogeneous environmental systems. The fractional spatial derivative $\frac{\partial^{\beta} C(x,t)}{\partial x^{\beta}}$ accounts for non-local transport, allowing particles to exhibit long jumps or Lévy flight behavior, which cannot be represented using classical second-order derivatives.The framework of model of transport anomalies gives a more universalized and physical consistent approach by enhancing both the temporal and spatial fractional operator as in the case of the Eq. (2). The formulation is especially applicable to environmental systems like ground water flow in fractured media, the movement of pollutants in heterogeneous soils and dispersion mechanism in complex natural settings.

3.3 Boundary and Initial Conditions

To guarantee a well formulated formulation of the transport problem as in the form of equations. There should be appropriate initial and boundary conditions, (1) and (2). The first condition involves the spatial distribution of the contaminant concentration at the beginning time t=0 and the boundary conditions involve the behavior of the system at the boundaries of the domain. In the current research, the first condition is taken to be a localized concentration field with a focus point inside the spatial field, which corresponds to an eventual emission of a source of contaminants. This can be mathematically expressed as:

$$C(x,0) = C_0(x)$$

where$C_0(x)$denotes the initial concentration profile. The boundary conditions are defined over a finite spatial domain$x \in [-L, L]$, where the concentration at the boundaries is assumed to approach negligible values due to dispersion effects. This can typically be written in Dirichlet boundary conditions:

$$C(-L,t) = 0 \quad C(L,t) = 0$$

These conditions indicate physically realistic situation in environmental systems where the concentration of the contaminant decreases sufficiently with the increasing distances between the source and the contaminants. Fig. 1 conceptually compares classical and anomalous transport models to their effects on the behavior of transport brought about by these initial and boundary conditions as demonstrated in Fig. 1. According to Fig. 1, the classical advection diffusion equation (Eq. (1)) yields a pronounced localised sharped, symmetric, Gaussian concentration curve which decays rapidly and spreads locally, reflecting localised spreading. By comparison, the fractional-order model (Eq. (2)) has a more extended distribution with higher tail areas, which are indicative of the long-range interactions and memory effects. In particular, the increased concentration values at greater distance in Fig. 1 are evidence of the non-local transport behaviour by the fractional spatial derivative and the decreased peak concentration is testimony of the role of time-memory by the fractional order parameter α. This is in agreement with anomalous transport observed in heterogeneous environmental systems including migration of pollutants in porous media and fractured formations groundwater. In such a way, the initial and boundary conditions imposed, along with the formulation used in a fractional order allows a more realistic representation of the dynamics of transport, which is demonstrated by the non-classical Gaussian profiles in Fig. 1.

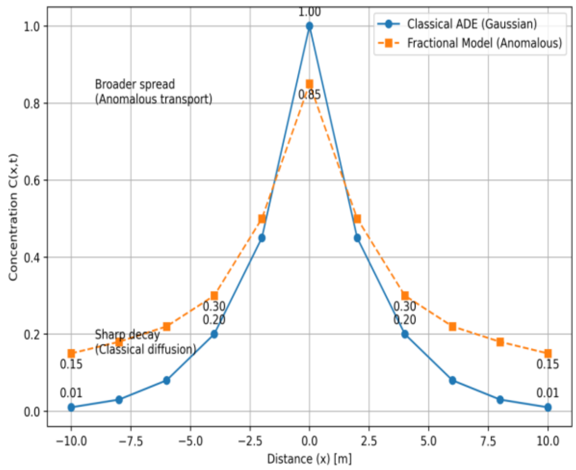

Figure 1. Classical vs Anomalous Transport Behavior: Conceptual Comparison of Gaussian Diffusion and Heavy-Tailed Distribution

4. Numerical Methodology and Simulation Framework

4.1 Discretization and Numerical Formulation

Most useful environmental systems do not have a closed-form analytic solution to the fractional-order transport model in the form of the equation presented in Eq. (2); thus, solution to the models will necessitate a numerical solution. The Grunwald- Letnikov (GL) approximation is used to discretize the fractional derivatives in this research because it is simple to use, and can be easily modeled using numerical techniques. A discrete convolution form can be used to approximate the Caputo time-fractional derivative, and it has the advantage of including the memory effect of past time delays. The GL approximation of the fractional time derivative is given as:

$$ {}^{C}D_t^{\alpha} C(x, t_n) \approx \frac{1}{\Delta t^{\alpha}} \sum_{k=0}^{n} (-1)^k \binom{\alpha}{k} C(x, t_{n-k}), \_\_\_\_(3) $$

whereΔt represents the time step size, and $\binom{\alpha}{k}$ denotes the generalized binomial coefficient defined for fractional orders. Such a formulation is an obvious problem of the non-locality of the fractional derivative since the present state is determined by all the past time steps. To spatial discretize, the finite difference method is used to estimate the fractional spatial derivative.The Riesz fractional derivative of order β is discretized using a symmetric difference formulation, given by:

$$ \frac{\partial^{\beta} C(x,t)}{\partial x^{\beta}} \approx \frac{1}{\Delta t^{\beta}} \sum_{m=-M}^{M} g_m^{(\beta)} C(x + m\Delta x, t), \_\_\_\_(4) $$

whereΔx is the spatial step size, and $g_m^{(\beta)}$ are the weighting coefficients associated with the fractional derivative approximation.And replacing the discretized equations. The fractional transport model (Eq. (2)) is then read into (3) and (4) resulting in a fully discrete numerical scheme. The resultant formulation can be used to calculate the values of concentration at the subsequent levels of time with a time stepping method. The grid point i and time level n discrete equation can be written as:

$$ C_i^n = f\left(C_i^{n-1}, C_{i\pm m}^{n-1}, \alpha, \beta, v, D\right), \_\_\_\_\_\_\_\_(5) $$

whereC_i^n denotes the concentration at spatial node i and time step$n$, and the function $f(\cdot)$represents the combined effect of advection, diffusion, and fractional operators.This numerical formulation converts the continuous fractional differential equation into a solvable algebraic system, to compute the anomalous transport dynamics efficiently. By doing so, the numerical solution is able to maintain both memory effects and non-local spatial interactions and this is vital in modeling the environmental transport phenomena accurately.

4.2 Numerical Algorithm and Implementation.

The fractional-order transport model has its numerical solution based on an iterative time-stepping algorithm, where the discretized equations obtained in the equations are integrated. (3)–(5). The general computational process involves the simultaneous setting of physical and numerical parameters, then the initial and boundary conditions are imposed on the specified spatial and temporal domain.

Initially, the model parameters, including the advection velocity v , diffusion coefficient D, fractional orders α and β, and discretization step sizes $\Delta x$and Δt, are specified. A computational grid is then established over the spatial domain, and the initial concentration distribution $C(x, 0)$ is assigned based on the problem formulation. Boundary conditions are also set at the domain boundaries to provide a numerical stability and physical consistency. The solution is time dependent and is iterative. The Caputo fractional derivative is calculated in every time step with the help of the Grunwald-Letnikov approximation that involves the input of all the earlier time levels. This leads to memory reliant behavior whereby the stored historical values of concentration have to be accumulated. At the same time the non-local transport effects are approximated by the discretization scheme chosen to estimate the spatial fragmental derivative, e.g. the use of the finite difference method.

Following the evaluation of fractional operators, the concentration field $C(x, t_{n+1})$ is computed at the next time level by combining the effects of advection, diffusion, and fractional dynamics. The algorithm then verifies the convergence criterion or the final simulation time that has been set.If the stopping condition is satisfied, the computation terminates; otherwise, the time step is updated $t = t + \Delta t$, and the process repeatsThe entire process of the numerical implementation can be described with the help of Fig. 2 that shows a step-by-step description of the algorithm, including the examination of the process of initializing, discretizing, choosing the iterative calculation, and assessing the convergence. The flowgraph shows how the fractional operators add memory effects and shows the iterative nature of the process that leads to the final solution.

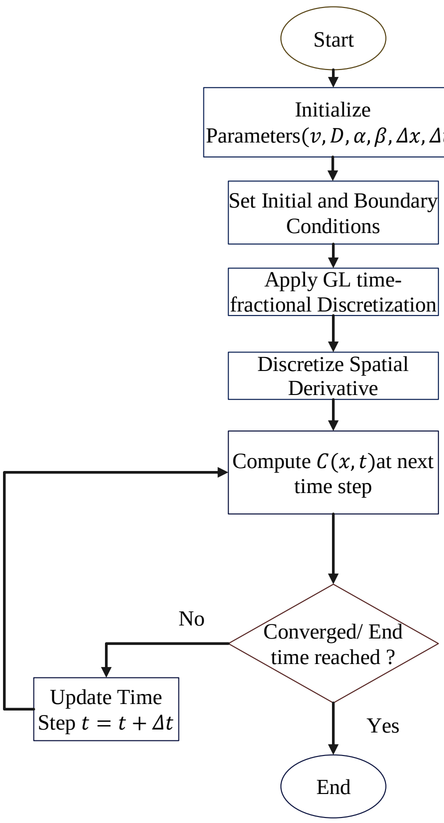

Figure 2. Numerical Solution Algorithm Flowchart for Fractional-Order Transport Model Implementation

4.3 Stability Analysis and Simulation Set up.

The stability and convergence of the proposed numerical scheme is the basis of its accuracy and reliability. The time-stepping algorithm in the current formulation deviates from the Grunwald-Letnikov approximation, adding to the convolution term a term that is history-dependent, thus a careful choice of discretization parameters is necessary to achieve numerical stability.The stability of the scheme is influenced by the choice of time step Δt, spatial step Δx, and the fractional orders αandβ. The discretization parameters are chosen in order to ensure stability of the numerical solution that remains bounded and not oscillated in the simulation domain.

Convergence of the numerical method is ensured by refining the discretization grid and verifying that the solution approaches a consistent profile as $\Delta t$and Δx decrease. A short examination of errors is carried out by comparing the results of solutions at consecutive grid resolutions by ensuring that the numerical scheme has stable and convergent behavior. The non-locality of the fractional derivatives makes the build-up of the past states to cause extra computational complexity but the discretization used ensures that the error propagation is within acceptable limits.

The simulation domain is defined over a finite spatial interval x $x \in [-L, L]$, where L is chosen sufficiently large to capture the dispersion behavior of the transport process. The time domain is fractionated into equivalent time steps and the time variation of the concentration field can be observed.The selection of model parameters, including the advection velocity v, diffusion coefficient$D$, and fractional orders αandβ, is based on representative environmental conditions to ensure physical relevance of the results.

The boundary and initial conditions outlined in Section 3.3 are introduced into the numerical system to ensure consistency of the mathematical model and a simulation environment. The initial concentration distribution is used to obtain the initial approximation of the iterative solution and the boundary conditions are used to make the concentration values physically significant at the boundaries of the domain. This combined strategy allows the numerical model to well model the anomalous transport effects and retain stability and physical realism.

5. RESULTS AND DISCUSSION

5.1 Model Validation

These are the advantages of the proposed model in terms of the fractional-order transport that are tested by a comparative test to the classical advection-diffusion equation (ADE).The simulation results obtained from both models under identical conditions are presented in **Fig. 3**, which illustrates the spatial distribution of concentration at a fixed time$t = 5$. As indicated in Fig. 3, the classical ADE model (Eq. (1)) gives a symmetric Gaussian concentration profile which has a sharp peak at the point of origin of the source and declines rapidly as the distance increases. This is what is happening as it is assuming the local diffusion and homogeneous media, in which the transport process is only controlled by short-range interaction. Concentration values are reduced exponentially with the center, which means that there is no extensive diffusion of the material being carried.

The behavior of the fractional-order model (Eq. (2)) in contrast has a much different transport behavior. The concentration profile in Fig. 3 corresponding to the previous has a wider concentration profile with less peak and higher tail regions. This implies that the fractional model is capable of capturing the long-range transport effects, thus enabling the particles to have a longer distance of transport than it was in the classical model. The fact that the higher concentration values are at larger distances shows the heavy-tailed property of the anomalous transport which cannot be modeled with the help of the integer-order derivatives. The difference between the two models is specifically notable in the tail of Fig. 3 when the fractional model still has non-negligible values of concentration, but classical model does not with low concentration levels in the tail. The non-local spatial operator and the time dynamics which are dependent on the memory due to the fractional derivatives are credited to this behavior. The fractional model gives a more realistic description of transport phenomena in heterogeneous environmental systems as a consequence.

Furthermore, the reduced peak concentration observed in the fractional model indicates a redistribution of mass over a wider spatial domain, which is consistent with sub-diffusive transport behavior governed by the fractional order parameter$\alpha < 1$corresponds. This validates the ability of the proposed model to capture both memory effects and anomalous spreading mechanisms. All in all, the findings in Fig. 3 indicate that the fractional-order model provides a significant contribution over the classical ADE in non-Fickian transport behavior modeling. The fact that the spreading, heavy-tailed distribution and improved physical consistency were enhanced provides evidence of the efficiency of this approach in case of environmental transportation.

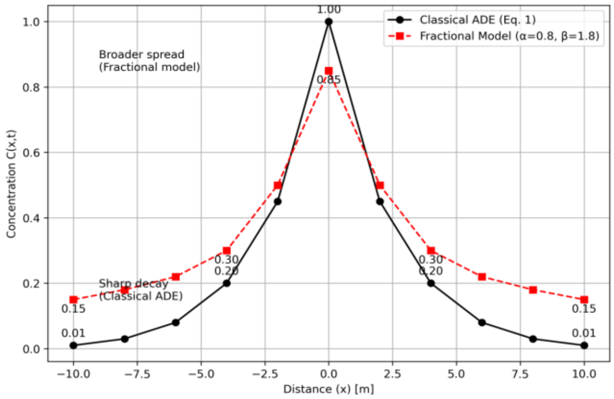

Figure 3. Comparison of Classical and Fractional Model Predictions at = 5: Validation of Anomalous Transport Behavior

5.2 Effect of Fractional Parameters

The influence of fractional-order parameters on transport behavior is analyzed to understand the sensitivity of the proposed model to variations in the temporal fractional orderα. The results are presented in **Fig. 4**, where concentration profiles are plotted for different values of αwhile keeping the spatial fractional order fixed at$\beta = 2$.As illustrated in **Fig. 4**, the value of α has a significant impact on the shape and spread of the concentration distribution. When$\alpha = 1.0$, the model reduces to the classical case, producing a sharp and narrow Gaussian profile with a pronounced peak at the center. This is indicative of no memory effects and is characteristic of normal Fickian diffusion.

As the fractional order decreases to $\alpha = 0.9$ and further to $\alpha = 0.8$, a noticeable in the transport dynamics is observed. Specifically, the concentration profiles become progressively broader, and the peak value decreases. This implies that the substance that is transported diffuses a greater spatial domain the stronger the impact of the memory effects. The wider dispersion indicates retention and delayed transport behavior, which are typical of the processes of anomalous diffusion. This effect is especially clear in the tail regions of Fig. 4 since lower values of 2alpha alpha lead to greater levels of concentration at greater distances. This phenomenon shows the increased persistence of particles in the system as a result of non-local temporal dynamics that the fractional derivative brings. This means that the system does not forget past states hence slower decay and longer spreading than in the classical case. The trends noted prove the fact that degree of memory in the transport process is controlled by the parameter α. Reduction of alpha enhances the memory effect leading to sub-diffusive behavior and non-Fickian characteristics of transport. This sensitivity analysis demonstrates how the fractional-order model can utilize this flexibility to achieve a broad variety of transport behavior, by simply adjusting the fractional parameters. Altogether, the findings in Fig. 4 show that the fractional-order model represents better transport dynamics than classical models. The model is capable of capturing many different anomalous regimes of transport by changing the fractional parameters, and has been used to describe a variety of different regimes in the environmental engineering field.

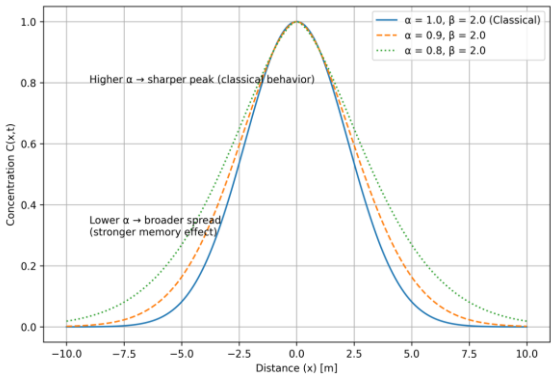

Figure 4. Influence of Temporal Fractional Order α on Transport Dynamics (with $\beta = 2$Fixed)

5.3 Environmental Case Study

A case study is taken to show how effectively the proposed fractional-order transport model can be applied to the pollutant transport in a groundwater system. Groundwater contamination is a very important environmental problem, in which the correct modeling of contaminant flow is vital to risk assessment and remediation design. Heterogeneous porous media, fractured geological structures, and complicated flow processes are typical features of such systems, resulting in non-Fickian transport behaviour.

The plume ontology: With the developed numerical framework of Section 4, the transport of a contaminant plume is modeled under realistic environmental conditions. The first one is a localized source of pollutants, and boundary conditions are applied to represent realistic flow behavior in the subsurface.The model parameters, including the advection velocity v, diffusion coefficient$D$, and fractional orders αandβ, are selected to capture the effects of medium heterogeneity and long-range interactions.The outcomes of the fractional model show that there is a very different pattern of transport in comparison to the classical predictions. Specifically, the contaminant plume is characterized by longer spreading and slower attenuation, which is also similar to anomalous transport of real groundwater systems. The fact that at greater distances the concentration is still high means that pollutants can travel greater distances than classical models are able to predict hence posing a greater risk of contamination to a larger area.

Also, memory effects have been captured by use of the temporal fractional derivative and this gives the model the ability to capture retention and trapping effects that are common in porous materials. It results in a more realistic model of behavior of contaminants particularly when transport is affected by adsorption, diffusion into immobile regions, or intricate transport paths. The results of the current case study illustrate the failure of classical advection diffusion models in environmental modeling and showcase the benefits of the fractional order method with regard to description of transport processes in the real world. The proposed model may be useful in helping environmental engineers to develop effective monitoring and remediation schemes because they will have better prediction of pollutant dispersion and persistence so they can use this information in creating appropriate strategies.

6. Comparative Analysis and Discussion.

In order to assess the efficacy of the suggested fractional-order transport model, we are carrying out comparative analysis regarding the classical advection-diffusion equation (ADE) in terms of major performance indicators such as accuracy of prediction, computational complexity and realistically explaining the physics. The summary of the comparison is presented in Table 1 that demonstrates the basic differences between the two modeling approaches.

Table 1. Comparative Analysis of Classical and Fractional Transport Models

Model	Accuracy	Complexity	Realism
Classical ADE (Eq. 1)	Low	Low	Poor
Fractional Model (Eq. 2)	High	Moderate	High

The classical ADE model, as shown in Table 1, proves not to be particularly accurate in predicting anomalous transport behaviour, mainly because of its dependence on local diffusion hypothesis and homogeneous system description. This weakness is evident in the validation findings in Section 5.1 (Fig. 3), which show that the classical model gives a steep-peaked Gaussian profile with a fast decay and does not take into consideration the influence of transport over long distances. Although the classical model has the advantage of being low-computation complex because it is formulated in a simple manner, it cannot capture real-world environmental transport processes.

Contrarily, the fractional-order model has a better performance based on both accuracy and realism. As shown in Fig. 3, the fractional model will be able to capture more extreme concentration distributions and heavy-tailed behavior, which is indicative of non-local interaction and memory-dependent dynamics. The sensitivity analysis in **Section 5.2 (Fig. 4)** further confirms that the model can adapt to varying transport conditions through the adjustment of fractional parameters αandβ, enabling it to represent a wide range of anomalous diffusion phenomena. This flexibility goes a long way in helping to improve the physical consistency of the model in case of complex environmental systems.

The better modeling of the fractional approach, however, is accompanied by a rise in the complexity of computation. Non-local operations are introduced by the use of a fractional derivative, and must accumulate past values between time steps, which complicates computations. However, as we shall see later in Section 4, the efficient numerical schemes like the Grunewald Letnikov approximation assure us that the computational needs are not too high to be viable in practice. On the whole, the comparative analysis suggests that the classical ADE model is a computationally efficient model that does not allow accurate representation of anomalous transport processes. The fractional-order model although being of moderate computational cost offers a much better representation of transport dynamics by including memory effects and non-local interactions due to space. These results verify that the suggested fractional modeling framework provides a stronger and more physical realistic framework to study the environmental transport phenomena.

7. CONCLUSION AND FUTURE WORK

This paper provided a fractional-order differential model of investigating abnormal transport conditions in environmental engineering systems. The classical advectional diffusion equation was known to be limited and to overcome this limitation, the use of the fractional derivatives included in the equations helped in developing memory effects and non-local spatial interactions. The numerical findings indicated that the fractional model can be applied to significantly enhance the accuracy of the transport dynamics especially in environments that are heterogeneous and in the classical models that do not consider the heavy tails and the long-range spreading behavior.

The validation analysis showed that the fractional model can describe the transport processes more accurately and in a more realistic way than the classical one. The increased concentration profiles and tail behavior in Fig. 3 point to the capability of the model in embracing non-Fickian diffusion behavior. Moreover, the sensitivity analysis given in Fig. 4 showed that the fractional parameters, especially the temporal order α\alphaα, is an important parameter in order to regulate the level of anomalous behavior, and allows the model to adapt to various regimes in transport.

Practically, the suggested framework has great benefits in environmental use like pollutant dispersion modeling, analysis of ground water contamination as well as risk assessment. The model can assist in making better decisions in terms of environmental monitoring and remediation strategies because it will give a more precise prediction of the contaminant migration and persistence.

Although the benefits of the fractional modeling approach are evident, it has more computational complexity and difficulty in estimating parameters. Consequently, the next step in research ought to be to generalize the proposed model to multi-dimensional systems to perform more realistic simulations of the environment. The integration of artificial intelligence and machine learning techniques for automatic estimation of fractional parameters αandβ represents a promising direction to enhance model adaptability and efficiency. Also, concrete implementation of the model can be enhanced by the creation of real-time simulation models and data-driven applications in future dynamic environmental monitoring systems.

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