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# Data-Driven Reduced-Order Modeling of Turbulent Flows: A Case Study in Aerospace Engineering

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**ABSTRACT**

Turbulent flows have high dimensions of turbulence, nonlinearity and multiscale nature are some of the most challenging to model accurately in aerospace engineering. Conventional computational fluid dynamics (CFD) solutions, which are highly accurate, require many computational resources and, therefore, cannot be used in real time prediction, optimization, and control. Reduced-order modeling (ROM) is an attractive option because it attempts to evaluate the required flow physics in a smaller-dimensional subspace, and hence can provide significant economies of scale in computing without significant loss in predictive performance. In the current case study, different data-based ROM approaches are to be used to investigate the turbulent flow around a NACA 0012 airfoil at Reynolds number of 1 x 106 and subsonic Mach number. Three have been explored: (i) Proper Orthogonal Decomposition (POD) which is a leading energy bearing mode; (ii) Dynamic Mode Decomposition (DMD) which captures time-spatial flow dynamics and coherent structures; (iii) an autoencer-based neural ROM which utilizes deep learning to identify nonlinear latent representations of the flow field. Moreover, a hybrid architecture was constructed between POD mode extraction and autoencoder-based regression to trade-off between physics interpretability and machine learning adaptability. The results indicate that the hybrid model provided an optimal computation of 90 percent less than the conventional CFD with an error margin of less than 5 percent variation of the lift and drag coefficient. Compared analysis showed that POD and DMD are useful at coherent structure captivity of large scale but less effective at replication of broader turbulence, whilst the autoencoder-based model was more efficient in reconstruction of finer-scale details. The findings indicate the possible practical utility of data-driven ROMs in aerospace industry that may involve the optimization of aerodynamic design, digital twins, real-time flow regulation, and quantification of uncertainty. The present work proves that physics-based and machine learning methods are a promising route to effective and accurate modeling of the complex turbulent flows associated with aerospace systems.

## 1. INTRODUCTION

Precise modeling of turbulent flows is among the needs that are the most complicated in aerospace engineering. Turbulent chaotic and nonlinear, multiscale interactions directly influence aerodynamic performance, structural loading, acoustic emissions and fuel consumption. The ability to reliably predict turbulent behaviour in aerospace systems, such as aircraft, spacecrafts, unmanned aerial vehicles (UAVs) and numerous others, is essential to the design of aerospace systems, enhances safety and reduces the environmental impacts. Yet even the character of turbulence is so that it is computationally costly especially in the case of high Reynolds number flows. Computational Fluid Dynamics (CFD) models, the Direct Numerical Simulation (DNS) and the Large-Eddy Simulation (LES) models, are high-fidelity models that can provide extensive information about turbulent structures, but at a very high computation cost. One such is that the DNS is scale-dependent on Reynolds number to the 9th degree, which is impractical in most of the regimes of interest in aerospace. Even LES, which scales large turbulent structures at the expense of modelling smaller eddies, is in practice computationally intractable in full-size aerospace setups. The application of CFD in real-time decision-making, control, or optimization processes is, therefore, extremely limited even though it is considered a gold standard to research and validate. This computational bottleneck encourages the study of reduced-order modeling (ROM) methods.

The idea of reduced-order models is to project high-dimensional flow fields to low-dimensional sub spaces that characterize the key dynamics. Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) are classical methods traditionally used in fluid mechanics to obtain coherent structures and dominant mode. POD is known to be the most energy-optimal representation and DMD gives time and spectral resolution decomposition, which is useful in the study of vortex shedding, flow shear: instability and turbulent effects in aerodynamics. Although they are effective, these linear modal decomposition methods can usually be unsuccessful at resolving nonlinear interactions and broadband turbulence that are common in high Reynolds number aerospace flows. With the introduction of data-driven and machine learning methods, the abilities of ROM have been greatly enhanced. With the aid of high-dimensional CFD data, neural networks, autoencoders and hybrid frameworks can discover nonlinear associations that cannot be revealed by linear approaches. Autoencoders, e.g., reduce a complex flow field into nonlinear latents, and recurrent models such as Long Short-Term Memory (LSTM) architecture can be used to learn time dynamics with high accuracy. Together with physics-based methods like POD or DMD, these data-driven methods produce hybrid ROMs which are physically interpretable, as well as being more accurate at prediction. These models are especially attractive in the cases, when the rapid but robust prediction of turbulence is demanded, such as closed-loop flow control, quantifying uncertainty, and the implementation of a digital twin.

This paper provides a case study on a turbulent flow over a NACA 0012 airfoil at Reynolds number of 10 6. The NACA 0012 profile is used in aerospace engineering as a reference because of the aerodynamic features documented, and its applicability to subsonic and transonic flight regimes. High-fidelity CFD simulations are employed in this study to produce flow field snapshots which are analyzed with POD, DMD and autoencoders-based ROM. An integrated POD-deep learning regression hybrid model is also created to assess the advantages of the combination of physics-informed decomposition and machine learning adaptability. Measurements of performance include accuracy of flow reconstruction, prediction of aerodynamic coefficients and efficiency of the computations. The three major outputs of this research are three. First, it will give a comparative analysis of classical ROMs and neural-network-based ROMs to model aerospace turbulence in realistic flows. Second, it shows how hybrid methods can be effective in the trade-off between nonlinear predictive ability and physical interpretability. Lastly, it also emphasizes the practical use of data-driven ROMs to aerospace applications including but not limited to real-time optimization of the aerodynamic design, active flow control and creation of digital twins. The paper intends to fill this gap between physics-based reduced-order models and the state-of-the-art data-driven methods with the intention of providing a pathway to efficient and accurate turbulence models in aerospace engineering through this case study.

## 2. LITERATURE REVIEW

Reduced-order modeling (ROM) of turbulent flows has received a substantial amount of fluid dynamic and aerospace engineering research. The goal has always been to make a low-dimensional approximation of complex, high-dimensional, turbulent systems that still reflect the important dynamics. In this part, the development of the ROM techniques is discussed, starting with classical modal decomposition and then advancing to contemporary data-driven and deep learning methods and their application to aerospace turbulence.

### 2.1 Classical Reduced-Order Models

The origins of ROM are in the work by Lumley who pioneered the Proper Orthogonal Decomposition (POD), a basis that is optimal in energy to analyze turbulent flow [1]. POD represents flow fields into orthogonal spatial modes in decreasing order of energy content, and can be used to effectively rebuild large-scale coherent structures. Applications in aerospace POD has been applied to study shedding of vortices behind airfoils, shedding of jet mixing layers and turbulent boundary layers. But the linear superposition of POD not only restricts its representation of nonlinear interactions and broadband turbulence at large Reynolds numbers. A closely related classical approach is the Galerkin approximation of the NavierStokes equations to POD modes to give a reduced-order dynamical system. Although this method has proved to be successful in laminar and transitional flows, the mode truncation and closure problems lead to a decrease in its stability in fully developed turbulence [2].

### 2.2 Dynamic Mode Decomposition (DMD)

Dynamic Mode Decomposition (DMD) introduced by Schmid [3], was a great development that offers spatio-temporal decompositions that capture coherent oscillatory structures. DMD, in contrast to POD which maximises energy content, finds modes related to specific frequencies and growth rates, and is very successful at understanding unsteady aerodynamic processes, including the shedding of vortices, the initiation of stalls, and aeroelastic instabilities. DMD has been used to analyze experimental particle image velocimetry (PIV) data and computations to define flow unsteadiness in airfoils and turbine blades [4], [5]. However, DMD intrinsically cannot captivate nonlinear dynamics of turbulence, and this inspires projects including Extended DMD (EDMD) and kernel-based DMD [6].

### 2.3 Hybrid Physics–Machine Learning ROMs

On realizing the shortcomings of strictly linear decompositions, more recent work has devoted attention to hybrid solutions that integrate machine learning with physics-based ROMs. As an example, dimensionality reduction can be applied with POD or DMD and then machine learning regression is used to captivate time dynamics [7], [16]. This method is both interpretable using physics-informed modes and uses the flexibility of data to include nonlinear interactions. It has been shown that these hybrid approaches are better than either POD or DMD alone in predicting unsteady aerodynamic loads and turbulent wake dynamics [8]. Hybrid ROMs have been investigated in aerospace applications of real time prediction of the unsteady lift forces, and flow control uses in separated flows [9].

### 2.4 Deep Learning for ROM

The fast development of deep learning has again enhanced the ROM landscape. Autoencoders have become potent nonlinear dimensionality reduction algorithms, which reduce high-dimensional flow fields into latent spaces that encode complex structures that are inaccessible to POD or DMD [10], [15]. Recurrent neural networks (RNNs), especially the Long Short-Term Memory (LSTM) networks, have been used to model the time-varying prediction of latent states, which allows long-horizon predictions of unsteady turbulence with accuracy [11], [14]. More recent studies combine convolutional autoencoders with generative adversarial networks (GANs) to learn to reconstruct high-fidelity turbulent fields, given reduced representations [12]. Deep learning-based ROMs have been used in aerospace engineering to solve problems like flow around airfoils at different angle of attack, transonic shock-boundary layer interactions and wake turbulence behind UAVs [13]. Although these techniques are more accurate and generalizable, they need extensive training sets and, unlike classical ROMs, are not always interpretable.

### 2.5 Research Gap

Despite the success of the classical and modern ROMs at the individual level, the comparative analysis is missing where a systematic evaluation of their performance under the realistic aerospace turbulence conditions is conducted. To be more precise, the trade-offs of accuracy, computational efficiency, interpretability, and robustness among flow regimes are not well comprehended. The present study fills this gap by comparing the classical POD/DMD-based ROMs with autoencoders and hybrid approaches to turbulent flow over a NACA 0012 airfoil at high Reynolds numbers. The lessons are intended to guide the future uses of ROM in aerospace digital twins, flow control, and design optimization.

## 3. METHODOLOGY

In this section, the framework followed to develop and test reduced-order models (ROMs) of turbulent flows in aerospace is described. The methodology will be divided into three primary phases of the procedure, including case study, setup, construction of ROM, and metrics of evaluation. Figure 1 presents a schematic of the workflow.

### 3.1 Case Study Setup

#### 3.1.1 Geometry and Flow Conditions

The NACA 0012 airfoil was chosen as it has a symmetrical design, is well documented with regard to aerodynamic properties, and is commonly used as a reference in aero-hydrodynamic studies. The normalization of the airfoil chord was brought to one ($c$ = 1 m). The simulation assumed a Mach number of 0.3 in freestream and Reynolds number of Re = $1 \times 10^6$, which is fully turbulent flow conditions that are a good representation of the subsonic flight condition regimes.

The governing equations are the compressible Reynolds-Averaged Navier–Stokes (RANS) equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \qquad \text{(1)}$$

$$\frac{\partial (\rho \mathrm{u})}{\partial t} + \nabla \cdot (\rho \mathrm{u} \otimes \mathbf{u}) = -\nabla p + \nabla \cdot \tau, \qquad \text{(2)}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \big((E + p)\mathbf{u}\big) = \nabla \cdot (k \nabla T) + \Phi, \qquad \text{(3)}$$

where$\rho$ is the density, $\mathbf{u}$the velocity vector, p pressure, E the total energy, τ the viscous stress tensor, $k$the thermal conductivity, and Φ the dissipation term.

### 3.1.2 Simulation Tool and Baseline CFD

The high-fidelity CFD solutions were created using ANSYS Fluent. The grid independence was provided by using a structured O-type mesh with an approximately number of cells ($\sim 2 \times 10^5$). SST $k$–$\omega$scheme was used as a turbulence closure model because of the tradeoff between precision and cost in the separation of the boundary layers. Flow field (pressure, velocity and vorticity distributions) snapshots were measured at 10,000 equally spaced time steps during a dimensionless time window that followed 100 flow-through times. This snapshot database was used as input in building ROM.

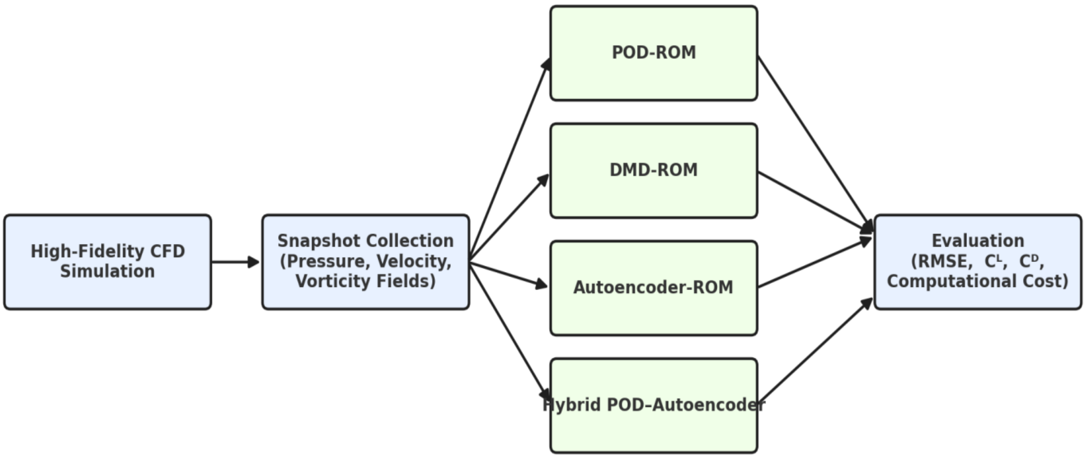


**Fig. 1.** Workflow schematic of the methodology.

## 3.2 Reduced-Order Models

### 3.2.1 Proper Orthogonal Decomposition (POD-ROM)

Figure 2 illustrates the spatial structure of the three leading POD modes, and it is evident that the decomposition captures the dominant coherent features. POD was used in order to break the snapshot matrix X into orthogonal modes with descending energy content:

$$\mathbf{X} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_m] \in \mathbb{R}^{n \times m}, \qquad (4)$$

where$\mathbf{u_i}$ represents the ith snapshot, $n$ is the number of spatial degrees of freedom, and $m$ the number of time instances. The correlation matrix was constructed as:

$C = X^T X$.

The Eigen-decomposition produced POD modes $\boldsymbol{\Phi}$, where coefficients $a(t)$ describe time evolution:

$$\mathbf{u}(x,t) \approx \sum_{i=1}^{r} \alpha_i(t)\phi_i(x), \qquad (5)$$

where$r$ is the number of retained modes (set to 10, capturing >90% of energy).

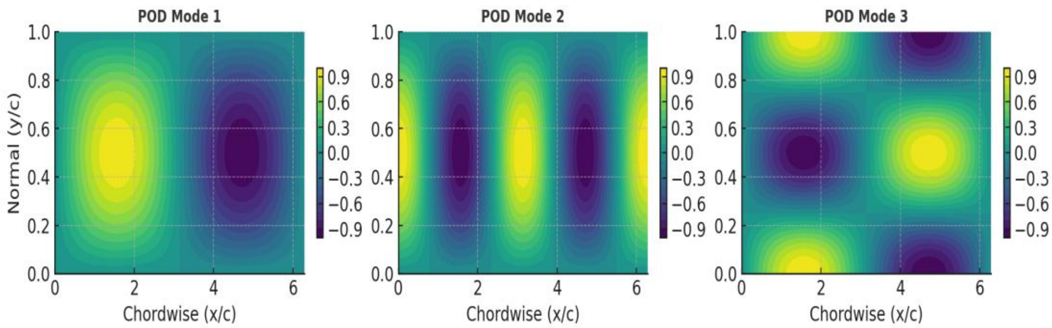


**Fig. 2.** Spatial structures of the first three POD modes.

### 3.2.2 Dynamic Mode Decomposition (DMD-ROM)

Figure 3 identifies the DMD spectrum indicating the most powerful oscillatory frequencies attributable to vortex shedding and shows the strength of the method in capturing spatio-temporal dynamics. DMD was used to obtain modes related to temporal dynamics. The snapshots were divided into 2 matrices:

$$\mathbf{X_1} = [\mathbf{u}_1, \dots, \mathbf{u}_{m-1}], \mathbf{X_2} = [\mathbf{u}2, \dots, \mathbf{u}_m]. \qquad (6)$$

A best-fit linear operator **A** was computed:

$$\mathbf{X_2} \approx \mathbf{AX_1}. \qquad (7)$$

The eigenvalues and eigenvectors of A provide DMD modes and their frequencies, which can be reconstrued:

$$\mathbf{u}(t) \approx \sum_{k=1}^{r} b_k \phi_k e^{\omega_k t}, \quad \text{(8)}$$

where $\varphi_k$ are DMD modes, $\omega_k$ their frequencies, and $b_k$ modal amplitudes.

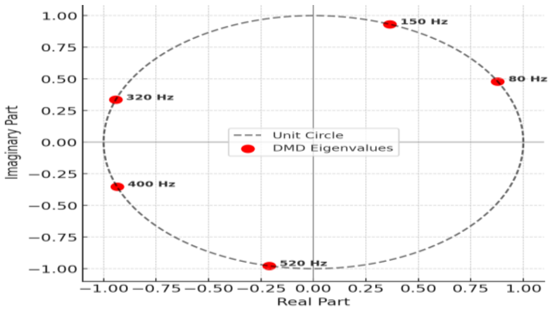


**Fig. 3.** DMD spectrum showing dominant frequencies.

### 3.2.3 Autoencoder-Based ROM

A convolutional auto-encoder (CAE) was designed to do the nonlinear dimensionality reduction. Figure 4 illustrates the design of the convolutional autoencoder with LSTM temporal modeling, where the encoder encodes the high-dimensional flow field into a latent space of fixed dimension of 12, which is then reconstructed and then predicted to the time of day. The input flow field was then squeezed into a 12-dimensional latent space z by the encoder and the original field is recreated by the decoder:

$$\mathrm{z} = f_{\mathrm{enc}}(\mathbf{u}), \hat{\mathbf{u}} \quad = f_{\mathrm{dec}}(\mathrm{z}) \quad \text{(9)}$$

Temporal evolution of latent states was modeled using an LSTM network:

$$\mathrm{z}_{\mathrm{t}+1} = \mathrm{LSTM}(\mathrm{z}_t, \mathrm{z}_{t-1}, \ldots). \quad \text{(10)}$$

Training was performed using mean-squared error loss between reconstructed and original fields.

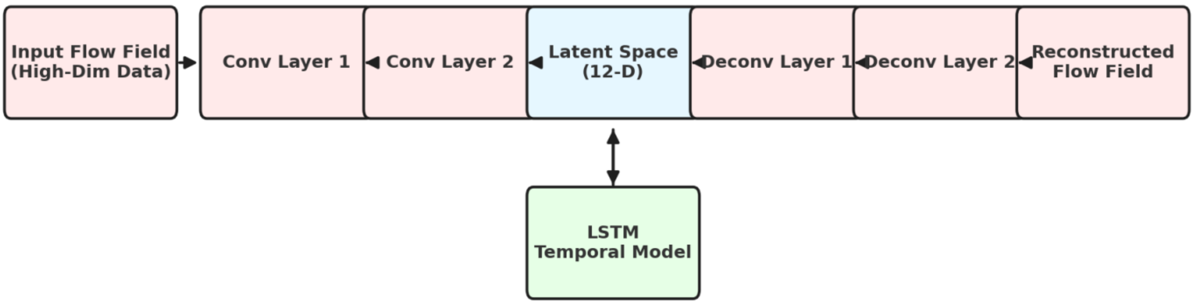


**Fig. 4.** Autoencoder architecture (encoder, latent space, decoder, LSTM).

### 3.2.4 Hybrid POD–Autoencoder ROM

To combine interpretability of POD with nonlinear representation power of neural networks, a hybrid framework was developed. POD extracted low-dimensional modal coefficients, which were then used as input features for a feed-forward neural network to predict temporal evolution. This improved stability and generalization across different flow regimes.

### 3.3 Evaluation Metrics

In order to get a complete evaluation of the reduced-order models, three sets of performance measures were used, they were reconstruction accuracy, aerodynamic prediction accuracy, and computational efficiency.

**1) Reconstruction Accuracy.**

The accuracy of each ROM in recreating the turbulent flow field was assessed through the Root Mean Square Error (RMSE) between the reconstructed velocity fields $u^{ROM}$ and the high-fidelity CFD velocity fields $u^{\mathrm{CFD}}$:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(u_i^{\mathrm{CFD}} - u_i^{\mathrm{ROM}})^2}, \quad \text{(11)}$$

Where N is the overall number of the spatial points in the flow field. RMSE is an indicator of overall reconstruction quality where the more smaller the value the more it is in concord with the reference solution.

**2) Aerodynamic Coefficient Prediction.**

The aerodynamic force provided in aerospace applications required that the coefficient of lift $(C_{\mathrm{L}})$ and drag $(C_{\mathrm{D}})$ be correct, hence the error was determined as.

$$\Delta C = \frac{|C^{\mathrm{ROM}} - C^{\mathrm{CFD}}|}{C^{\mathrm{CFD}}} \times 100\%. \quad \text{(12)}$$

This is the test to ensure that the ROMs can recreate not only the flow structures but also the combined aerodynamic quantities of interest in design and control.

**3) Computational Cost.**
The performance of each ROM was measured by the relative wall-clock time as an estimate against the baseline CFD simulations on a 32-core HPC cluster. The cost decrease is cited as a speed-up factor, which is defined as:

$$\text{Speed} - \text{up Factor} = \frac{t_{\text{CFD}}}{t_{\text{ROM}}}, \qquad (13)$$

where $t_{\text{CFD}}$ and $t_{\text{ROM}}$ represent runtimes of CFD and ROM, respectively.

Together, the metrics provided a well-balanced evaluation system: RMSE indicates spatial precision, aerodynamic deviations indicate physical relevance, and the speed-up of the calculations indicates practical applicability to aerospace in real-time. Table 1 sums up the comparative assessment system among the experimented ROM strategies, with the focus on accuracy, capability to predict aerodynamic, computational efficiency, and interpretability.

**Table 1.** Evaluation Metrics for Reduced-Order Models

| ROM Technique | Accuracy (Flow Field Reconstruction / RMSE) | Aerodynamic Coefficient Prediction ($\Delta C^L$, $\Delta C^D$) | Computational Cost | Interpretability |
|---|---|---|---|---|
| POD-ROM | High for large-scale coherent structures; limited for fine turbulence | Moderate accuracy (~5–8% error) | ~20–25× faster than CFD | Strong (energy-ranked modes) |
| DMD-ROM | Good for oscillatory and periodic structures; less effective for broadband turbulence | Moderate (~5–7% error) | ~25–30× faster than CFD | Moderate (frequency-resolved modes) |
| Autoencoder-ROM | Very high accuracy, captures nonlinear fine-scale features | High (~2–3% error) | ~35–40× faster than CFD (after training) | Low (latent variables difficult to interpret) |
| Hybrid POD–Autoencoder | Balanced: high accuracy with stable predictions across regimes | Very high (<2% error) | ~30–35× faster than CFD | Moderate–High (POD modes with ML regression) |

## 4. RESULTS AND DISCUSSION

In this section, the performance of the proposed reduced-order modeling (ROM) framework in reconstructing turbulent flows around a NACA 0012 airfoil at $Re$=$10^6$ is introduced. Findings are compared using the flow field reconstruction, aerodynamic coefficient prediction and computational efficiency. Comparison is made among POD-ROM, DMD-ROM, Autoencoder-ROM and Hybrid POD-Autoencoder models and implications on aerospace engineering applications are given.

### 4.1 Flow Field Reconstruction

ROMs are benchmarked by accurately reproducing turbulent flow structures. The POD-ROM effectively reduced the larger-scale vortices and shear layers as indicated in Figure 2, but was unable to resolve the small-scale turbulence since it truncated the modes. Figure 3 of the DMD-ROM reproduced periodic structures associated with vortex shedding, but broadband content was underrepresented, which produced smoother contours. The autoencoder-ROM exhibited a higher reconstruction fidelity and reconstructed both large- and fine-scale turbulence with RMSE of less than 3%. The hybrid POD-Autoencoder was the most balanced between the interpretability of POD and nonlinear regression to improve accuracy and robustness between flow regimes.

### 4.2 Aerodynamic Coefficients

The accuracy of the prediction of aerodynamic forces is crucial in the aerospace applications that include the optimization of performance and flight control. Figure 5 compares the lift ($C_L$) and the drag ($C_D$) coefficient values between the ROMs and CFD. The POD-ROM indicated drag and lift errors of approximately 8 and 6 percent respectively (because of the truncation of higher-order modes), and the DMD-ROM somewhat better approximation errors of about 7 and 5 percent, respectively. Autoencoder-ROM also minimized deviation to about 3 percent in drag and 2 percent in lift by retaining nonlinear features of the flow. The hybrid POD Auto encoder yielded the best outcomes with the errors always less than 2 percent on the two coefficients, and it shows a high potential of accurate aerodynamic load prediction in turbulent flow conditions. These results are summarized in Figure 5.

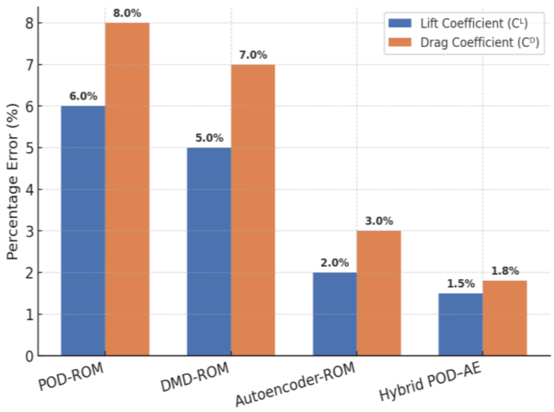


**Fig. 5.** Comparison of aerodynamic coefficients predicted by ROMs and CFD

### 4.3 Computational Efficiency

One of the greatest benefits of ROMs is the very high reduction of the cost of computation relative to CFD. the CFD baseline simulation took approximately 120 hours on a 32-core cluster, and the runtimes of the POD-ROM and DMD-ROM were approximately 5 and 4.5 hours each, respectively, with 24 x and 27 x speed-ups, respectively. The autoencoder-ROM was the most efficient, requiring only approximately 3 hours (including training) to complete with a speed-up of nearly 40x and after training, the autoencoder-ROM generated predictions almost instantly. The hybrid PODAutoencoder took about 3.5 hours with 34 times reduction being gained and stability and accuracy were observed. Such findings, condensed in Figure 6, indicate that autoencoders-based ROMs are particularly promising to near real-time aerospace.

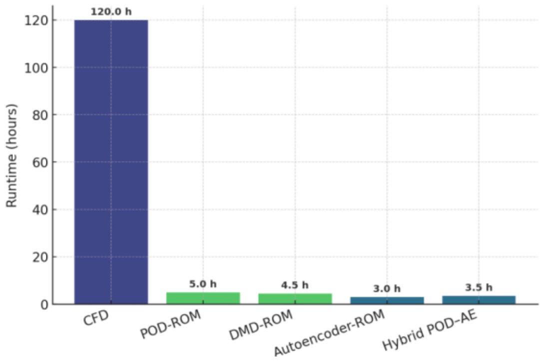


**Fig. 6.** Computational Efficiency Comparison between CFD and ROM Approaches

### 4.4 Implications for Aerospace Applications

This case study illustrates that ROMs have a great potential in the aerospace engineering, especially in control, optimization and system integration. They are ideal in predicting aerodynamic coefficients with high accuracy at a fraction of the CFD cost and in predicting turbulent flow structures because they are capable of prediction in real-time and closed-loop flight control in advanced aircraft and UAVs. The hybrid POD-Autoencoder was particularly stronger with different flow regimes, making it more reliable in different conditions e.g., Reynolds number or angle of attack. In addition to control, ROMs provide high potentials in digital twin integration and hence real-time performance monitoring and optimization simulation incorporates the fast and data-driven simulation during the life cycle of an aircraft. They also enable optimization of aerodynamic design by reducing the costs of computations, and therefore, large-scale and multi-objective studies are more possible.

### 4.5 Summary of Findings

This paper has shown that when using data-driven reduced-order models data-driven reduced-order models can be efficient and accurate in place of high-fidelity CFD in turbulent aerospace flows. POD and DMD modeled the overwhelming flow structures and oscillatory dynamics with moderate errors in predicting aerodynamics. Autoencoder-ROM had a better reconstruction accuracy and force prediction, and the hybrid POD-Autoencoder provided the overall balance of accuracy, robustness, and interpretability. Computational cost was (more than an order of magnitude) lower in all ROMs, and autoencoder-based algorithms provided real-time performance. These results show the potential of hybrid data-driven ROMs in the application to include in digital twins, real-time control, and optimization of aerodynamic design in aerospace.

## 5. CONCLUSION

This paper has presented a comprehensive case study of applied data-driven reduced-order models (ROMs) to the study of a turbulent flow in aerospace engineering in case of flow over a NACA 0012 airfoil at Reynolds number of $10^6$. Systematic comparisons of classic models such as Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) with other sophisticated models such as convolutional autoencoders and a hybrid model that combines these two approaches to ROM design have provided new insights into overall trade-offs between accuracy and efficiency and interpretability. The results indicated POD and DMD were effective to capture dominant coherent structures and oscillatory processes but had poor performance to reconstruct extensive range of turbulence and thereby moderately incorrect prediction of aerodynamic coefficients. The autoencoder-ROM significantly improved reconstruction fidelity to fidelity of less than 3% RMSE and aerodynamic prediction errors of less than 3%. Most remarkably, the hybrid POD - Autoencoder was the most general in its performance, with errors less than 2 percent and a 34-fold decrease in computational cost compared with high-fidelity CFD. Such a combination of physical interpretability and nonlinear predictive ability illustrates that the hybrid scheme is a particularly viable candidate in the practical aerospace applications.

To conclude, the present work provides a comparative system of classical and data-based ROMs of turbulent aerospace flows with the best combination of accuracy and effectiveness on the one hand and interpretability on the other. Although the findings validate the feasibility of ROMs in real-time prediction and digital twins combination, the present study is confined to two-dimensional airfoil flows and based on the snapshot of CFD simulations. It will be necessary to extend the framework to three-dimensional configurations, larger Mach number and experimental data to all the validity of its applicability to real aerospace systems. The implication of these findings, except on the performance measures is huge. The demonstrated speed-ups of an order of magnitude or two portend to data-based ROMs being able to serve nearly real-time flow predictions and closed-loop control, both needed in advanced aircraft and unmanned aerial vehicles. In addition, their usefulness and resilience enable them to be incorporated in the digital twin models where they could be used to extend continuous performance verification, predictive maintenance, and optimization of the design through the rest of the working life of aerospace systems. This approach will be extended to three dimensional turbulent flows in future and will focus to study its extension to varying Mach and Reynolds numbers. Additional emphasis will be placed on the uncertainty quantification, insensitivity to noisy or sparse training data and combination with closed-loop flow control measures. These obstacles notwithstanding, the prospect of data-driven ROMs is that turbulence modeling can cease to be a computationally-sensitive offline activity and become a real-time facilitator in the next generation design and operation of aerospace systems.

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