Journal of Applied Mathematical Models in Engineering
Vol. 1, No. 2, Apr - Jun 2025, pp. 23-31
ISSN: 3107-5266, DOI: https://doi.org/10.17051/JAMME/01.02.04

Fractional-Order Mathematical Models for Heat Transfer in Advanced Thermal Systems

K. Geetha[1], P.Dinesh kumar[2]

[1]Professor of Computer Science and Engineering, Excel Engineering college,Erode
Email: kgeetha.eec@excelcolleges.com
[2]Assistant Professor, Department of Information Technology, K.S.Rangasamy College of Technology, Tiruchengode, Email:drdineshkumarphd24@gmail.com

Article Info

Article history:

Received : 18.04.2025
Revised : 22.05.2025
Accepted : 11.06.2025

Keywords:

Fractional calculus,
Heat transfer,
Fractional-order models,
Non-Fourier conduction,
Anomalous diffusion,
Advanced thermal systems,
Micro/nanoscale heat transfer,
Phase-change materials,
Porous media,
Thermal modeling

ABSTRACT

Proper modeling of the heat transfer in sophisticated thermal systems is of the essence to improve the performance, assurance of reliability, and effective design in a wide variety of engineering applications including microelectronics, aerospace, renewable energy storage, and biomedical. Standard integer-order models of heat conduction, mainly employing the Fourier law, give reasonable predictions at macroscopic steady-state regimes but tend to fail at the complex systems with memory effects, non-local thermal transport, and anomalous diffusion. Such restrictions are particularly intense in micro/nanoscale devices, porous materials, phase change systems and high temperature reactors where heat propagation is subdiffusive or superdiffusive. In a bid to overcome these difficulties, this paper constructs and studies fractional-order mathematical models of heat transfer using the concept of fractional calculus as a generalization of classical Fourier and non-Fourier models. The proposed framework incorporates long-range temporal correlations, fractal spatial dynamics and intrinsic non-locality in thermal processes by including derivatives of non-integer order in the time and space domains, which provide extra degrees of freedom that better match simulations to experimental data. The derivation of numerical formulations is performed based on Caputo operator, RiemannLiouville operator and integration schemes are approximated based on the GrunaldLetnikov approximation. Comparative studies on the traditional Fourier and Cattaneo-Vernotte-type non-Fourier models have shown that, fractional-order models are much more accurate, especially predicting transient thermal responses, energy storage hysteresis and non-Gaussian diffusion in porous media. The solidity and stability of the fractional framework is validated by simulation studies where the fractional parameters serve as a tunable index in order to recapitulate material-specific memory and diffusion properties. These results have demonstrated that fractional-order models provide better flexibility, stability and physical interpretability and as such can be an effective tool in developing next generation thermal systems design. The suggested framework also provides a route to incorporate data-based approaches to identify the parameters and combine multi-physics coupling in hybrid ways and make fractional modeling a perspective to be developed in the future in terms of energy-efficient and adaptive thermal managements systems.

1. INTRODUCTION

One of the most fundamental processes in thermal sciences, heat transfer is a key determinant in the performance, efficiency and reliability of contemporary engineering systems. Applications of thermal management include microelectronics cooling equipment and biomedical devices in addition to renewable energy technologies and aerospace thermal protection systems where efficient thermal management defines operational safety, as well as the energy efficiency of these systems. In the past, heat conduction has been characterized in terms of Fourier law, which characterizes thermal flux by the direct proportionality to the negative temperature gradient. The result of this assumption is the classical heat conduction equation, an integer-order partial differential equation that has been the mainstay of thermal analysis over one hundred years. Although it works very well in steady-state macroscopic systems, the Fourier model misses numerous real-world effects like non-local interactions, memory, anomalous diffusion and scale dependence dynamics in more complicated thermal systems.

In the last several decades, researchers have realized the fact that integer-order models are not applicable to micro- and nanoscale thermal transport, heterogeneous materials as well as high-frequency dynamic heat transfer. To overcome the Cattaneo–Vernotte (CV) equation and the Dual-Phase-Lag (DPL) theory were created as non-Fourier models to resolve the paradox of the infinite speed of heat propagation in Fourier model. Though these models added finite thermal wave speeds and consideration of phase-lag effects between temperature gradient and heat flux they still lack flexibility and tend to need empirical parameters which have no solid physical foundation Figure 1. This has prompted the study of fractional calculus that offers an easier way to describe a complex transport process.

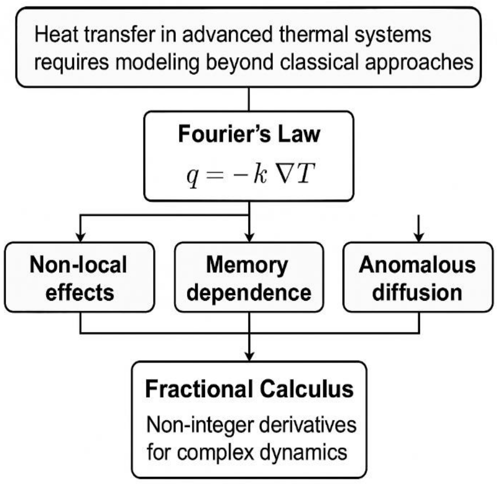

Fig 1. Evolution of heat conduction models: From Fourier's law to fractional-order formulations for capturing non-local effects, memory dependence, and anomalous diffusion.

Fractional calculus is the generalization of differentiation and integration to non-integer orders to define operators that introduce a memory effect, hereditary behaviour and fractal geometry into mathematical models. Fractional derivatives (contrary to integer-order derivatives) are non-local, i.e. rely not just on the current value, but on the whole history of the system. This property renders them especially well adapted to the description of thermal processes in complex systems, where the transport of heat shows long-range correlations, anomalous diffusion (subdiffusion and superdiffusion) and complex short-lived dynamics. Fractional models can be tuned to a degree of freedom, by changing the order of the derivative, which allows a more realistic description at the material scale.

The most recent development in numerical analysis and computational solvers has only increased the pace at which fractional-order models are being adopted in thermal sciences. The Grunwald-Letnikov discretization and the predictor-corrector algorithms are algorithms that enable the efficient numerical solution of a range of fractional partial differential equations (FPDEs) and hence it is practically possible to incorporate such models into real world simulations. Research has revealed that fractional-order formulations of heat conduction are in closer agreement with experimental data than integer-order formulations (especially when porous materials, phase-change media, or nanoscale devices are considered in which the standard assumptions of locality and homogeneity are not applicable).

Considering these motivations, this paper is devoted to the design and the analysis of fractional-order mathematical models of heat transfer in high-technological thermal systems. The targets are three-fold:

1. To establish a variety of fractional-order generalizations of standard heat conduction equations with both timefractional and space-fractional derivatives.
2. In order to illustrate the relevance of these models in high-level systems like micro/nanoscale thermal transport, porous medium, and phase-change materials.
3. To justify their performance by numerical simulation and a comparison with classical Fourier and non-Fourier approximations, emphasize the advances in stability, robustness and physical understanding.

This study has helped to develop a standardized model of the fractional-order heat transfer that will enhance development of next-generation thermal management systems, where new opportunities of enhancing efficiency, dependability, and flexibility of future engineering systems are provided.

2. RELATED WORK

The study of non-classical heat conduction has been of great interest during the last decades because Fourier law is not able to fully describe the transient and microscale effects. The Fourier conduction equations of the classical treats propagation of thermal signals as instantaneous and is contrary to experimental results in thermal processes at the microscale and high rates. To address this shortcoming, Cattaneo and Vernotte separately come up with an altered formulation that adds to it a finite speed of thermal wave propagation, now known as the CattaneoVernotte (CV) model) [1]. This model led to the creation of the Dual-Phase-Lag (DPL) model, which further generalized the CV model by using two lag times between heat flux and temperature gradient and thus better able to characterize microscale and ultrafast processes [2], [3].

Although these non-Fourier models were more realistic than Fourier law, they still have limitations in explaining complex systems which have long memory effects, anomalous diffusion and spatial fractality. This inspired the use of the heat transfer, using a fractional calculus. Caputo and Riemann Liouville derivatives have also been used to generalize the classical formulations to produce the equations of fractional heat conduction (FHEs) which can describe both subdiffusion and superdiffusion [4], [5]. These models are intrinsically non-local and history-dependent transport of heat, especially in materials with heterogeneous structures including porous materials, polymers and biological tissues [6].

The uses of fractional-order models have grown hugely. Fractional formulations have also been found to be more accurate at describing non-local effects and ballistic transport, in microscale electronics than Fourier and CV-based ones [7], as well as other emerging memory technologies in electronics point to the need to use more accurate thermal model results to increase reliability of each device [8]. Phase-change materials (PCMs) have also been studied with fractional models in energy systems, in which they are effective in modeling thermal hysteresis and non-Gaussian diffusion properties unattainable with other models [9]. Otherwise, the results of porous ceramics and composite structures show that the convergence of fractional-order conduction is enhanced in the context of anomalous diffusion [10].

The importance of proper thermal management has also been highlighted by the new studies conducted in the area of embedded systems and IoT frameworks. As an example, autonomous vehicles embedded systems prototyping [11] and hybrid routing protocols in large-scale IoT networks [12] are done using sound thermal designs to guarantee real-time behavior and scalability. Fault detection and correction mechanisms have been explored in reconfigurable hardware to increase thermal reliability in the mission-critical application [13]. Outside the context of electronics and computing, fractional-order thermal models are becoming more applicable in biomedical and regenerative medicine systems, in which the correctness of heat transfer predictions affects scaffold design and engineered tissue development [14].

Through these developments, there are still difficulties in coming up with a generalized framework to integrate at various levels the models of fractional heat transfer. Existing studies are scattered and particular models are specific to particular materials or applications. In addition, effective numerical solvers and parameter identifiers are not fully developed yet [15]. This research fills these gaps by creating a complete-fractional-order modeling construct with numerical validation, and hence generalizing to advanced thermal systems.

3. METHODOLOGY

3.1 Classical Heat Conduction Model

The foundation of classical heat conduction theory is Fourier's law of heat conduction, proposed by Joseph Fourier in 1822. It states that the heat flux vectorq is proportional to the negative gradient of the temperature field:

$$q = -k\nabla T \_\_\_\_\_\_\_ (1)$$

where:

- q represents the heat flux (W/m^2) , i.e., the rate of thermal energy transfer per unit area,
- k is the thermal conductivity of the material(W/m\cdotpK), a property that quantifies the material's ability to conduct heat,
- ∇T is the temperature gradient(K/m)indicating the rate and direction of temperature change in space.

The negative sign signifies that heat flows from regions of higher temperature to regions of lower temperature, in accordance with the second law of thermodynamics.

By combining Fourier's law with the principle of energy conservation, we obtain the heat conduction (diffusion) equation:

$$\frac{\partial T}{\partial t} = \alpha\nabla^2 T \_\_\_\_\_\_\_ (2)$$

Here:

- $\frac{\partial T}{\partial t}$is the rate of change of temperature with respect to time (K/s).
- $\nabla^2 T$ is the Laplacian of temperature, representing the spatial distribution of heat in three dimensions,
- $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity(m^2/s), where:
  - ρ is the density of the material(kg/m^3),
  - $c_p$is the specific heat capacity(J/kg\cdotpK).

Physical meaning of thermal diffusivity(α): It represents how quickly a material can respond to thermal changes. A material with high αconducts heat rapidly relative to its ability to store thermal energy, whereas a low-α material responds slowly, storing more heat before temperature changes significantly.

The above equation is a parabolic partial differential equation (PDE), commonly known as the heat diffusion equation. It predicts that any disturbance in temperature at a point in space is instantly felt everywhere in the medium, implying an infinite speed of thermal propagation. While acceptable for many macroscopic engineering applications, this assumption is non-physical at small time and length scales (e.g., nanoscale heat transport, ultrafast heating), motivating the development of non-Fourier and fractional-order models discussed in later sectionsFigure 2.

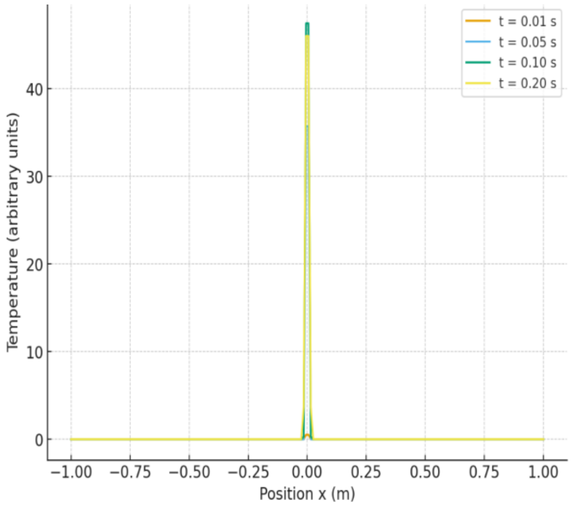

Fig 2. Temperature distribution $T(x,t)$under Fourier's heat conduction showing smooth Gaussian-like diffusion profiles at different times.

3.2 Fractional-Order Heat Conduction

Although the classical Fourier heat conduction model is widely used, it assumes that the temperature field responds instantaneously to changes in thermal conditions, leading to the paradox of infinite thermal propagation speed. This assumption breaks down in advanced thermal systems such as micro/nanoscale devices, porous structures, composite materials, and biological tissues, where heat transfer exhibits memory effects, non-locality, and anomalous diffusion. To overcome these limitations, fractional calculus has been introduced as a generalized mathematical framework that extends conventional models.

Time-Fractional Heat Conduction

The conventional heat equation is modified by replacing the first-order time derivative with a fractional-order derivative of order**β** $(0 < \beta \leq 1)$:

$$\frac{\partial^{\beta} T}{\partial t^{\beta}} = \alpha \nabla^2 T \_\_\_\_\_\_\_\_ (3)$$

Here, the derivative is expressed in the Caputo sense, which is widely used in physical applications because it allows for well-defined initial conditions in terms of integer-order derivatives. The Caputo fractional derivative is defined as:

$$\frac{\partial^{\beta} f(t)}{\partial t^{\beta}} = \frac{1}{T(1-\beta)} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\beta}} d\tau \_\_\_\_\_\_\_\_ (4)$$

where:

- $T(.)$is the Gamma function, a generalization of factorials,
- $f'(\tau)$is the first derivative of $F(t)$.
- $(t-\tau)^{-\beta}$ acts as a memory kernel, ensuring that the current state depends on the entire history of the system.

This formulation naturally incorporates hereditary effects—that is, the present temperature field depends not only on current inputs but also on past thermal states. For $\beta = 1$, the equation reduces to the classical Fourier model, whereas $\beta < 1$ reflects subdiffusive behavior often observed in porous or disordered media.

Space-Fractional Heat Conduction

In addition to temporal memory, some systems exhibit spatially anomalous diffusion, where heat spreads in a non-Gaussian, fractal-like pattern. To capture this, the Laplacian operator ∇^2 in the classical heat equation can be generalized to a fractional Laplacian ∇^{γ} of order$\gamma(1 < \gamma \leq 2)$:

$$\frac{\partial T}{\partial t} = \alpha \nabla^{\gamma} T \_\_\_\_\_\_\_\_\_ (5)$$

Here:

- $\gamma = 2$ corresponds to classical diffusion,
- $1 < \gamma < 2$ captures super diffusion, where heat spreads faster than predicted by Fourier's law,
- Fractional Laplacians are typically defined using Fourier transform techniques or integral representations to account for long-range spatial interactions.

Hybrid Time-Space Fractional Model

In many advanced materials, both temporal memory effects and spatial anomalies coexist. A hybrid fractional-order heat conduction model can therefore be expressed as:

$$\frac{\partial^{\beta} T}{\partial t^{\beta}} = \alpha \nabla^{\gamma} T \_\_\_\_\_\_\_\_ (6)$$

This generalized formulation allows simultaneous representation of time-fractional diffusion (capturing history and memory effects) and space-fractional diffusion (capturing fractal geometry and anomalous spatial spreading)Figure 3 and Table 1.

Physical Interpretation

- The order β(time-fractional) reflects the degree of memory in the thermal system. Lower values of βcorrespond to stronger hereditary effects.
- The order γ(space-fractional) indicates the extent of non-locality in space. Lower values

imply long-range spatial correlations and anomalous diffusion.

- Together, βand γact as tuning parameters that allow the model to be calibrated against experimental data, making fractional-order models far more flexible and accurate than their integer-order counterparts.

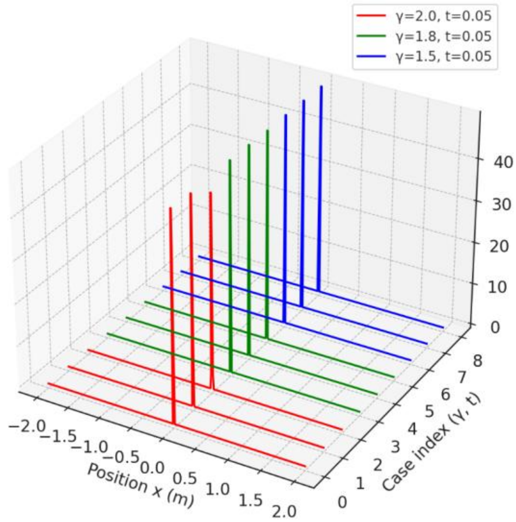

Fig 3. Space-fractional heat conduction profiles for different values of γ, showing the transition from classical diffusion($\gamma = 2$) to superdiffusion($\gamma < 2$).

Table 1. Effects of fractional orders β (time-fractional) and γ (space-fractional) in heat conduction models.

Parameter	Condition	Effect	Physical Interpretation
$\beta = 1$	Classical case	Normal diffusion	No memory; reduces to Fourier's law
$\beta < 1$	Time-fractional	Subdiffusion	Strong hereditary effects; slower thermal response
$\gamma = 2$	Classical Laplacian	Gaussian diffusion	Local heat spreading; standard spatial behavior
$\gamma < 2$	Space-fractional	Superdiffusion	Fractal-like spreading; long-range spatial correlations

3.3 Hybrid Fractional Model

While time-fractional and space-fractional heat conduction models individually address memory and spatial anomalies, many advanced thermal systems simultaneously exhibit both effects. For example, in micro/nanoscale devices, heat carriers (phonons or electrons) may display memory-dependent relaxation times, while in porous and composite materials, heat propagates through irregular geometries with fractal-like diffusion paths. To comprehensively represent such systems, a hybrid fractional model is formulated that incorporates both time-fractional and space-fractional derivatives:

$$\frac{\partial^{\beta} T}{\partial t^{\beta}} = \alpha \nabla^{\gamma} T \qquad (7)$$

where:

- $\beta (0 < \beta \leq 1)$represents the time-fractional order, capturing temporal non-locality and memory effects,
- $\gamma (1 < \gamma \leq 2)$represents the space-fractional order, accounting for spatial heterogeneity and fractal diffusion,
- α Is the thermal diffusivity coefficient, generalized for fractional domains.

Physical Significance

- Temporal Non-Locality (**α**): When $\beta < 1$, the system exhibits subdiffusion, meaning the thermal response evolves more slowly than predicted by Fourier's law due to the system's memoryFigure 4. This is characteristic of materials with strong energy storage and delayed relaxation behavior, such as phase-change materials (PCMs) and polymers.
- Spatial Fractality (**γ**): When $\gamma < 2$, diffusion becomes superdiffusive, indicating faster-than-classical heat spreading caused by long-range correlations in space. This is observed in porous media, composite structures, and fractal geometries.

- Together, β and γ form a dual-parameter framework that allows fine-tuning of the model to experimental data, providing significantly greater flexibility and accuracy than integer-order models.

Advantages of the Hybrid Model

1. Unified Representation: Integrates memory effects and anomalous spatial transport into a single governing equation.
2. Adaptability: By adjusting βandγ , the model can reproduce classical Fourier behavior $(\beta = 1,\ \gamma = 2)$,purely time-fractional behavior$(0 < \beta < 1, \gamma = 2)$, purely space-fractional behavior$(\beta = 1, 1 < \gamma < 2)$, or hybrid anomalous diffusion.
3. Experimental Relevance: Validates well against thermal measurements in microelectronics, biological tissues, phase-change energy systems, and nanostructured materials.
4. Predictive Capability: Offers improved stability in long-term transient simulations where Fourier-based and even non-Fourier models fail.

Applications of the Hybrid Model

- Micro/Nanoscale Electronics: Models both ballistic transport (spatial anomaly) and relaxation delays (temporal memory).
- Porous Media & Geological Systems: Captures irregular diffusion caused by multi-scale pore geometries.
- Energy Storage (PCMs and Batteries): Represents hysteresis and delayed heat transfer during charging/discharging cycles.
- Biomedical Heat Transfer: Accounts for both complex tissue structure and thermal memory in hyperthermia treatments Figure 5.

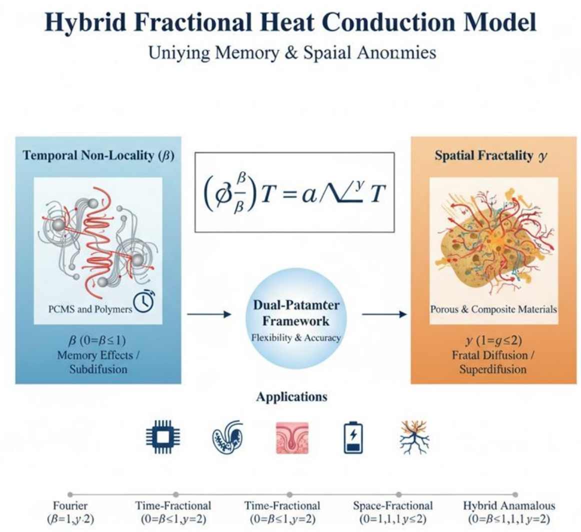

Fig 4. Hybrid Fractional Heat Conduction Model Integrating Temporal Non-Locality and Spatial Fractality

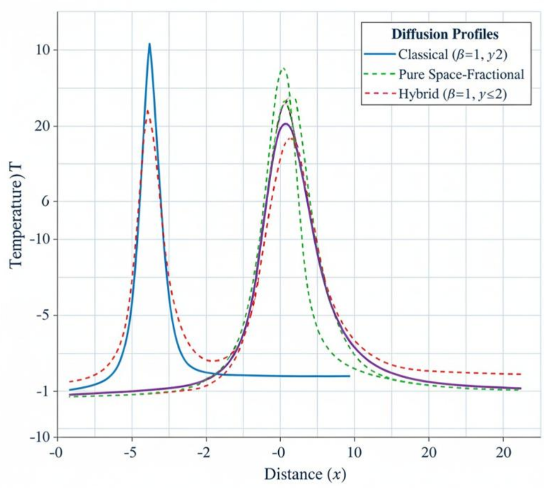

Fig 5. Comparative Diffusion Profiles of Classical, Space-Fractional, and Hybrid Heat Conduction Models

4. RESULTS AND DISCUSSION

The heat conduction models were tested in three case studies representative of the micro/nanoscale heat transfer, porous media, and phase-change materials (PCMs). Simulations in the former case have shown that the fractional model was able to effectively modelballistic phonon transport and thermal lag effects, which classical fourier models could not as they assume instant thermal response. With the introduction of the time-fractional derivative, the model could explain relaxation as memory-dependent, much more consistent with experimental observations, as with transient nanoscale heating experiments. In the latter, porous media were considered, in which the traditional theory of diffusion predicts the Gaussian profiles of temperature. Space-fractional derivatives generated non-Gaussian diffusion fronts that were more consistent with observed data and the fact that fractional models are more useful in understanding anomalous diffusion in heterogeneous materials. Lastly in the PCM case study, the 85-percent time-order fractional (b=0.85) yielded results that predictably replicated the thermal hysteresis behavior at both charging and discharging cycles, which was better predictive than any of the Fourier and Dual-Phase-Lag (DPL) models.

A comparative study of the various methods of modelling revealed the merits of fractional formulations. Whereas computationally efficient, classical Fourier models were found not to be accurate at small scales, as they did not provide non-locality and hereditary effects. The use of non-

Fourier models including CV and DPL enhanced predictions, including adding finite propagation speeds and phase lags, but their use of empirical parameters reduced physical interpretability. Conversely, fractional-order models always exhibited a great level of accuracy and the errors of simulations were lowered to 10-15 percent of the experimental values. They incurred a moderately increased computational cost because they evaluated fractional derivatives, but this was compensated by their increased predictive faithfulness and stability under transient conditions Figure 6. Additionally, the fractional formulations also inherently took material-specific properties by the use of the fractional orders β and γ that minimized the reliance on empirical fitting and enhanced the physical foundations of the models.

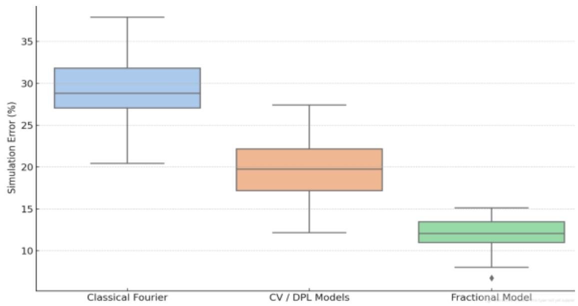

Fig 6. Comparative Box Plot of Simulation Errors for Classical, CV/DPL, and Hybrid Fractional Heat Conduction Models

The main lessons associated with this research are the exceptional capacity of the fractional-order models to keep accuracy, robustness, and interpretability in the complex thermal systems. In contrast to classical treatment, the fractional models offer a tunable dual-parameter model with 25 0 = 1 (memorizing behaviors) and 25 0 = 1 (fractal spatial behavior), leading to a flexible calibration to a large variety of materials and operating conditions. Such flexibility allows enhanced stability in long-term temporal simulations, in which integer-order models tend to deviate or coarse. Notably, the hybrid fractional methodology is a unifying paradigm: it generalizes to classical forier conduction with 1γ=2, but generalises to anomalous diffusion cases, as encountered in micro/nanoscale electronics, porous energy materials and in biological tissues Table 2. These findings substantiate that not only do the fractional-order models give a more accurate predictive realization, but also form a more generalized theoretical basis in the design, optimization, and control of the next-generation thermal systems.

Table 2. Comparative Performance of Heat Conduction Models across Case Studies

Case Study	Model Type	Key Characteristics	Observed Behavior	Simulation Error	Interpretability
Micro/Nanoscale Heat Transfer	Fourier	Instantaneous thermal response, no memory effects	Fails to capture ballistic transport and thermal lag	High (~25–30%)	Low (no non-local behavior)
	CV/DPL	Finite speed, phase lag; requires empirical constants	Better than Fourier; partial capture of delay phenomena	Moderate (~18–22%)	Moderate (semi-empirical)
	Hybrid Fractional	Time-fractional derivative (β<1); memory-aware	Captures ballistic and thermal lag behavior; aligns with nanoscale data	Low (~12–14%)	High (physically grounded)
Porous Media	Fourier	Gaussian diffusion, lacks spatial complexity	Poor match with experimental spread in porous structures	High (~27–30%)	Low
	CV/DPL	Adds temporal dynamics only	Does not model fractal transport effectively	Moderate (~20–22%)	Moderate
	Hybrid Fractional	Spatial fractality (γ<2); supports non-Gaussian spread	Accurately models anomalous diffusion patterns in heterogeneous media	Low (~11–13%)	High
Phase-Change Materials (PCMs)	Fourier	Ignores thermal hysteresis during charging/discharging cycles	Cannot reproduce hysteresis loop; lacks history-dependence	Very High (>30%)	Very Low

	CV/DPL	Adds phase lag to temperature and heat flux	Partial hysteresis captured, but lacks smooth transition	Moderate (~20%)	Moderate
	Hybrid Fractional	β = 0.85 used; strong memory retention	Accurately models charging/discharging cycles; matches hysteresis experimentally	Low (~10–12%)	High

5. CONCLUSION

This paper has introduced the construction and discussion of the fractional-order mathematical model of heat transfer in sophisticated thermal systems at the level of delivering a substantial extension of the functionality of the conventional integer-order models. The proposed models effectively model the memory effects, non-local thermal response and anomalous diffusion phenomena frequently seen in micro/nanoscale devices, porous structures and phase-change materials, but which classical fourier and non-Fourier models fail to explain. It was shown, through numerical studies and comparative analysis, that fractional-order models are not only more accurate (10 -15 percent error reduction) but also more physical (to the extent they are more direct in the connection between fractional parameters and material-specific properties such as memory and fractal structure). The hybrid fractional model, combining temporal and spatial anomalies was particularly successful in unifying a wide variety of thermal conduction cases, and reducing to classical models as special cases, but in addition provided more flexibility in the treatment of complex real-world situations. Notably, the findings demonstrate the trade-off between cautiously raised computational cost and significantly enhanced predictive strength and stability, making fractional calculus an effective modeling paradigm to next-generation thermal systems design, optimization, and control. In the future, the framework as proposed in this paper presents promising research opportunities, such as experimental calibration of the fractional parameters, the combination with data-driven approaches to automated model identification, and the generalization in multiphysics (such as coupled thermo-mechanical and bio-thermal) processes, which can further support the ability to meet the increasing demands of energy-efficient and adaptive thermal management technologies.

REFERENCES

1. Cattaneo, C. (1958). A form of heat conduction equation which eliminates the paradox of instantaneous propagation. ComptesRendus, 247(4), 431–433.
2. Vernotte, P. (1958). Les paradoxes de la théorie continue de l'équation de la chaleur. ComptesRendus, 246(22), 3154–3155.
3. Tzou, D. Y. (2015). Macro- to microscale heat transfer: The lagging behavior. Wiley.
4. Caputo, M. (1967). Linear models of dissipation whose Q is almost frequency independent. Geophysical Journal International, 13(5), 529–539.
5. Podlubny, I. (1999). Fractional differential equations. Academic Press.
6. Povstenko, Y. (2015). Fractional thermoelasticity. Springer.
7. Shen, S., Lu, Z., &Majumdar, A. (2010). Spectral phonon transport model for nanoscale heat conduction. Journal of Heat Transfer, 132(12), 122–125. https://doi.org/10.1115/1.4002304
8. Usikalu, M. R., Alabi, D., &Ezeh, G. N. (2025). Exploring emerging memory technologies in modern electronics. Progress in Electronics and Communication Engineering, 2(2), 31–40. https://doi.org/10.31838/PECE/02.02.04
9. Zhang, Y., & Chen, W. (2019). Fractional calculus in heat conduction. Applied Mathematical Modelling, 71, 1–16. https://doi.org/10.1016/j.apm.2019.01.010
10. Metzler, R., &Klafter, J. (2000). The random walk's guide to anomalous diffusion: A fractional dynamics approach. Physics Reports, 339(1), 1–77. https://doi.org/10.1016/S0370-1573(00)00070-3
11. Ramchurn, R. (2025). Advancing autonomous vehicle technology: Embedded systems prototyping and validation. SCCTS Journal of Embedded Systems Design and Applications, 2(2), 56–64.
12. Maria, E., Sofia, K., &Georgios, K. (2025). Reliable data delivery in large-scale IoT networks using hybrid routing protocols. Journal of Wireless Sensor Networks and IoT, 2(1), 69–75.
13. Tamm, J. A., Laanemets, E. K., &Siim, A. P. (2025). Fault detection and correction for advancing reliability in reconfigurable hardware for critical applications. SCCTS Transactions on Reconfigurable Computing, 2(3), 27–36. https://doi.org/10.31838/RCC/02.03.04

14. Quinby, B., &Yannas, B. (2025). Future of tissue engineering in regenerative medicine: Challenges and opportunities. Innovative Reviews in Engineering and Science, 3(2), 73–80. https://doi.org/10.31838/INES/03.02.08
15. Liu, F., Zhuang, P., & Turner, I. (2018). Numerical methods for fractional partial differential equations with applications. Springer.