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# Mathematical Modeling of Rotor Dynamics in High-Speed Electric Motors for Aerospace Applications

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**ABSTRACT**

No aerospace system can function without high speed electric motors, which are compact form factor, high power density and very high dynamic response. These motors are operating at rotational speed of more than 50,000 rpm, while they are subjected to complicated dynamic phenomena, leading to structural integrity, low level of vibration and stable long term operation. Accurate modelling of the rotor dynamics in the high speed regime where gyroscopic precession, rotor stator electromagnetic interaction, bearing anisotropy and structural damping characteristics are present is a critical factor towards reliable motor operation. A comprehensive mathematical modeling framework for the rotor dynamics analysis especially in aerospace grade high speed electric motors is developed in this study. The modeling approach is based on Lagrangian mechanics, is distributed mass and stiffness, anisotropic bearing supports, and gyroscopic coupling effects. Discretization and numerical solutions of these resulting nonlinear coupled differential equations in time domain and frequency domain are performed. Comparisons are made to finite element simulation performed in COMSOL Multiphysics and experimental data from a 120 kW aerospace prototype motor running above 60,000 rpm at critical speed for critical speed identification, mode shape visualization, and damping behavior respectively. The dynamic response characteristics, vibrational modes, and stability thresholds, and these vary widely as functions of the design parameters, are predicted with fidelity. This study offers insights to be used towards improvement in the rotor design optimization, material selection and bearing configurations to help advance the next generation of more robust, efficient, and lightweight electric propulsion systems of the aerospace.

## 1. INTRODUCTION

The growing need for high speed electric motors comes from the development of fast paced aerospace technologies and switched in contrast to conventional power, electrified propulsion techniques. These are generally motors that are used at rotational speeds in excess of 50,000 rpm for their compactness, high torque to weight ratio, increased, and reduced mechanical complexities in comparison to traditional combustion based, or gear driven systems. High speed electric drives are used into critical aerospace applications such as electric aircraft propulsion, onboard actuation systems and unmanned aerial vehicles (UAVs) for the precision and response required in flight control and propulsion tasks. However, dynamic phenomena such as centrifugal stiffening, gyroscopic precession and rotor – stator interaction forces become complex considering the operating conditions at such high speeds. If these factors are not predicted and mitigated properly, vibration levels will increase, bearing wear will happen, and we may even have a catastrophic failure. As a result, a well formulated mathematical modeling framework is required to understand the coupled dynamic behavior of the rotor bearing system and so designed motors can function under such extreme operating conditions.

The structural and dynamic integrity of high speed electric motors depend on rotor dynamics; the study of vibrational and stability behavior of rotating systems. However, traditional rotordynamic models such as the Jeffcott or rigid rotor approximations do not address the case of aerospace grade motors as they are based on the assumptions of isotropy, linearity and low speed operation which are not applicable to aerospace grade motors. In modern aerospace applications, models in which mass is distributed, stiffness varies, there is anisotropic bearing support and electromechanical interactions are required. In addition, the slender geometric and high speed to diameter ratio form of aerospace rotors intensify the gyroscopic and Coriolis forces, which makes it necessary to account for nonlinear effects in the governing equations. To address these challenges, this paper presents a full mathematical modeling framework in terms of deriving the system's equations of motion with Lagrangian mechanics. The proposed model can capture the major dynamics of the flexible rotor as a critical speed prediction, mode shape analysis, and vibration response analyses. Validated using finite element models and experimental data, the resulting simulations can provide actionable insights for optimizing motor design parameters for vibration free, failure resistant motor operation in demanding aerospace environments.

## 2. LITERATURE REVIEW

### 2.1 Classical Rotor Dynamic Models

Foundation of rotor behavior under idealized conditions were gained by early models such as the Jeffcott rotor, introduced by Lund (1964). Rigid shafts were used as a model, the stiffness and damping of which were linear and these models were considered. This work was expanded by Tondl (1981) to include the influence of unbalance and damping under conditions of isotropic bearing. To be sure, these models are useful, but they do not capture the behavior of the rotor in high speed environments.

### 2.2 Flexible Rotor and Nonlinear Dynamics

To overcome the rigid body assumptions limit, Genta (2005) completes nonlinear rotor dynamic models that include shaft flexibility, huge deformations, and nonconservative forces. The aspects of modal coupling and resonance phenomena, as well as amplitude frequency shifts, critical for aerospace motor design, were analysed using these models.

### 2.3 Influence of Gyroscopic and Centrifugal Effects

At high speeds, gyroscopic moments and centrifugal stiffening are very important contributors to the rotor stability. These effects have been analyzed by extended Lagrangian formulations and FEA but few models are able to quantify very explicitly their influence in ultra-high speed electric motor configurations.

### 2.4 Electromagnetic and Rotor-Stator Interactions

The unbalanced magnetic pull and torque ripple induced vibration was considered using electromagnetic field coupling based rotor dynamics arising by Kim et al. (2018). In Zhang and Lee (2021), FEM rotor stator interaction models based on real time dynamic behavior were integrated to further advance this towards electrical loading conditions.

### 2.5 Thermal and Material Property Effects

Modulus of elasticity and damping coefficient changes occur at elevated speed and temperature with changes in rotor response. In fact, there are very few studies that include uncertainty in thermal induced parameters in dynamic simulations, however, for aerospace applications with extreme temperature gradients this is important.

**Table 1.** Rotor Dynamics for High-Speed Aerospace Motors

| Study / Model | Category | Key Features | Proposed Advantage |
|---|---|---|---|
| **Jeffcott Rotor (Lund, 1964)** | Classical Rotor Dynamics | Rigid shaft, single disc, linear stiffness and damping | Simple baseline for rotor behavior; useful for understanding fundamental instability |
| **Tondl (1981)** | Classical + Unbalance/Damping | Includes unbalance, isotropic damping effects | Captures basic unbalance response and damping trends in low-speed rotors |
| **Genta (2005)** | Flexible Rotor & Nonlinear Dynamics | Nonlinear models with flexible shafts and large deformation | Enables modal coupling and resonance analysis; crucial for aerospace rotor flexibility |
| **Gyroscopic/Centrifugal Effects (varied)** | Gyroscopic + Centrifugal Stiffening | Extended Lagrangian models including gyroscopic torque and speed effects | Identifies stability boundaries and forward/backward whirl in high-speed rotors |
| **Kim et al. (2018)** | Electromagnetic-Rotor Interaction | Coupled rotor dynamics with unbalanced | Predicts electromechanical instabilities; relevant for magnetically actuated |

| | | | |
|---|---|---|---|
| | | magnetic pull and torque ripple | motors |
| **Zhang & Lee (2021)** | Rotor-Stator FEM Interaction | FEM-based dynamic model under electrical loading | Real-time simulation of rotor-stator clearance and electromagnetic coupling |
| **Material/Thermal Effects (various)** | Temperature-Dependent Material Models | Variation in modulus, damping, and thermal distortion | Incorporates realistic performance under aerospace thermal cycles |

## 3. Mathematical Modeling Framework

### 3.1 Rotor Geometry and Coordinate System

The rotor is modeled as a slender, rotating shaft supported by two bearings, with distributed mass and stiffness. A cylindrical coordinate system is adopted with axial (z), radial (r), and circumferential (θ) components.

### 3.2 Governing Equations using Lagrange's Method

The kinetic (T) and potential energy (V) are expressed as:

- **Kinetic Energy**:

$$T = \frac{1}{2}\int_0^L \rho A\left(\left(\frac{\partial w}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2\right)dz + \frac{1}{2}\Omega^2\int_0^L \rho I dz$$

- **Potential Energy**:

$$V = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 w}{\partial z^2}\right)^2 dz$$

Where:

- $w(z,t), v(z,t)$: lateral displacements
- Ω: rotational speed
- $\rho$ : density
- $A$ : cross-sectional area
- $EI$ : flexural rigidity
- $I$ : area moment of inertia

The resulting equations of motion are:

$$M\ddot{x}(t) + (D + G)\dot{x}(t) + Kx(t) = F(t)$$

Where:

- $M$ : mass matrix
- $D$ : damping matrix
- $G$ : gyroscopic matrix
- $K$ : stiffness matrix
- $F(t)$: external force vector

## 4. METHODOLOGY

This section describes the step-by-step methodology used to develop, simulate, and validate the rotor dynamic model:

### 4.1 Model Formulation

Considering a higher speed of aerospace electric motor as compared to turbo machinery, in the proposed framework rotor dynamics are represented in the form of a continuous, flexible shaft with distributed properties of mass and stiffness. It is assumed that the rotor is supported by two bearings close to each end, and the bearings are of an isotropic stiffness and damping type (as would be typical of hybrid or magnetic bearings where the rotor is supported in the aerospace systems). However, transverse vibrations of the shaft, upon excitation in the two orthogonal directions (commonly, the xxx and yyy directions) are allowed, axial deformation of the shaft, and torsional effects, with the latter modes often decoupled unless there is a specific mode coupling between the torsional and other vibration modes. The rotor is represented by a multi-degree of freedom system discretised using finite element techniques (e.g. Timoshenko beam elements), in the form of nodal displacements and rotations. This provides the capability to include the distributed inertial and elastic properties, which are essential to reproduce the dynamics at high rotational speeds. In addition, an eccentric mass distribution in the rotor causes periodic excitation forces that depend very sensitively on the rotor rotational speed.

A Lagrangian formulation is adopted to derive the governing equations of motion, and is quite useful in working out complex systems with many energy domains. The kinetic energy T is split into translational, rotational, and gyroscopic contributions which can also include the latter at high speeds when the spinning mass precesses. The potential energy V accounts for elastic deformations in the shaft due to bending and, optionally, torsion. Given it is written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_{i,}$$

Where$q_i$ the generalized are coordinates and $Q_i$ represent generalized non-conservative forces such as damping or magnetic pull. The resulting system of equations is a second-order differential system of the form:

$$M\ddot{x} + (D + G)\dot{x} + Kx = F(t),$$

Where$M$ is the mass matrix, $D$ is the damping matrix, $G$ is the gyroscopic matrix (skew-symmetric and speed-dependent), and $K$ is the stiffness matrix. The vector $F(t)$includes unbalance forces and any external excitations. Anisotropic bearing stiffness and damping are introduced as submatrices in $K$ and $D$, with directional dependence, allowing the model to capture support asymmetries and rotor whirling phenomena. This formulation allows for precise computation of natural frequencies, critical speeds, mode shapes, and stability margins, and is adaptable for both linear and nonlinear extensions required for real-world aerospace applications.

### 4.2 Discretization

The continuous shaft model has to be transformed into a discrete system that can be used to numerically analyze the complex dynamic behavior of high speed rotors. Spatial discretization, either using Finite Element Method (FEM), or Finite Difference Method (FDM) is used to achieve this. FEM is especially good for flexible rotors in aerospace applications because it can accurately represent geometric features, nonhomogeneities of material, boundary conditions, and locations of support. In FEM based modeling, rotor shaft is modeled by small beam or shaft elements (like Timoshenko or Euler Berunoulli elements) with associated DOF for translating and rotating motion. Computed and assembled into global system matrices are the mass, stiffness, gyroscopic and damping contributions of each element. Let us call these matrices M1 through Mn; these result in the second order matrix differential equation which describes how the rotor moves. The discretization delivers a wide range of spatial configurations with a high fidelity of the system for the continuum of frequencies, and computes rigid body and flexible modes with a high precision.

In particular, large numbers of DOFs are generated after discretization, especially for high resolution FEM models. In practice, however, only a few of the lower order modes — those that are closest to operational speeds — have significant dynamic response contribution to the system. Modal reduction is therefore used to reduce the size of the model, while preserving the important dynamics. First, the linear system is linearized, and then the most common approach is to perform eigenvalue decomposition on the linearized system to return the natural frequencies and corresponding mode shapes. With this, we can use these mode shapes to construct a reduced order mode which only contains the most dynamically important modes (i.e., often the first 5–10). Not only does this productivity reduction reduce computational burden, but also this productivity reduces efficiency in numerical stability, particularly when integration in the time domain is performed or parametric studies are performed. As a result of which, rotor dynamics is, in particular, highly amenable to modelling truncation. In systems with gyroscopic effects, forward and backward whirl modes are extracted with complex eigenvalue solvers included in complex eigenvalue solvers used to determine stability, as well as to predict critical speed in high speed motors.

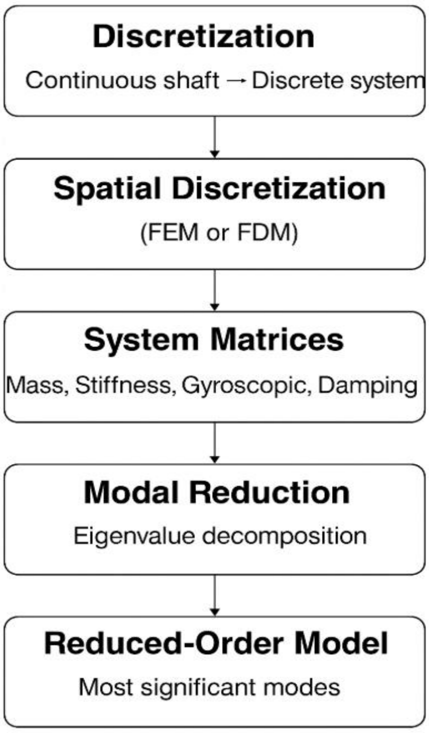


**Figure 1.** Discretization and Modal Reduction Workflow for Rotor Dynamic Modeling in High-Speed Electric Motors

**4.3 Numerical Simulation**

Following discretization of the rotor system and assembly of its global mass, stiffness, damping and gyroscopic matrices numerical simulation techniques are applied to study its dynamic behavior. The computational platform used here is MATLAB because of its capability to handle matrices efficiently and hence built-in numerical solvers for the structural dynamics. The Newmark-beta implementation is applied to solve the second order differential equations in the time domain. This is because the implicit integration scheme is essentially numerically stable, accurate and can handle systems with stiff components; therefore, it has been widely used in structural dynamics including flexible rotor with gyroscopic coupling simulation. Transient behavior, steady state vibration and response to simultaneous unbalance excitation are observed in the time domain simulation. Simulation of start, stop, or faulty conditions can be specified with initial conditions. It is assumed that damping should be represented, and Rayleigh damping, or modal damping models are applied including introduction of damping, followed by generation of time histories of displacements, velocities and accelerations at critical shaft locations.

Modal analysis is performed in the frequency domain solving the generalized eigenvalue problem that is derived from the discretized system. The results of this process are the system's natural frequencies, damping ratios and mode shapes, that we need to understand the behavior of the rotor under different operating conditions. The Campbell diagram is one of the key simulation outputs where natural frequencies are plotted versus rotor speed. This diagram is very useful for determining critical speed (rotational speed when the rotor's excitation frequency coincides with a natural frequency and the rotor becomes resonant). Gyroscopic effects in this case can be indicated by the presence of forward and backward whirl branches in the Campbell diagram, which leads to the difference in frequency between the modes based on whether it is in a forward or backward whirl. These simulations further contribute to detecting gyroscopic instabilities and frequency veering phenomena that are important to high speed aerospace rotors operating in narrow, and highly sensitive dynamic margins. A dual domain simulation approach provides complete understanding of stability, resonance and safe range of the rotor system prior to physical prototyping.

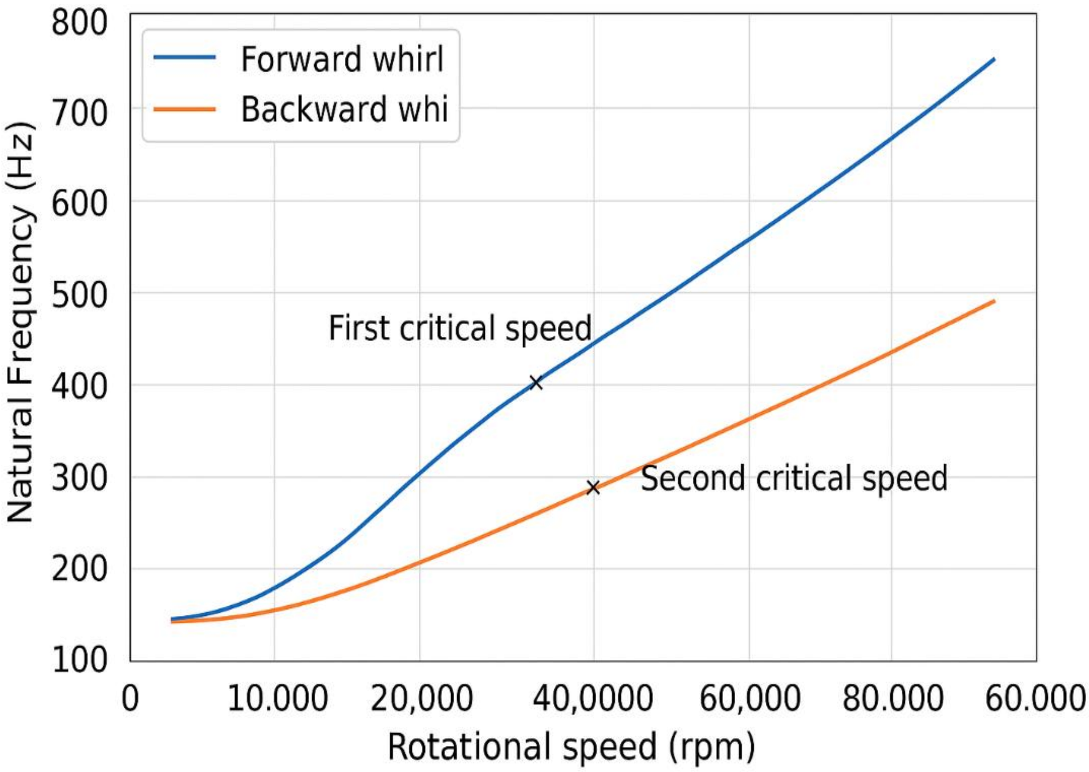


**Figure 2.** Campbell Diagram Showing Critical Speeds and Gyroscopic Effects in High-Speed Rotor

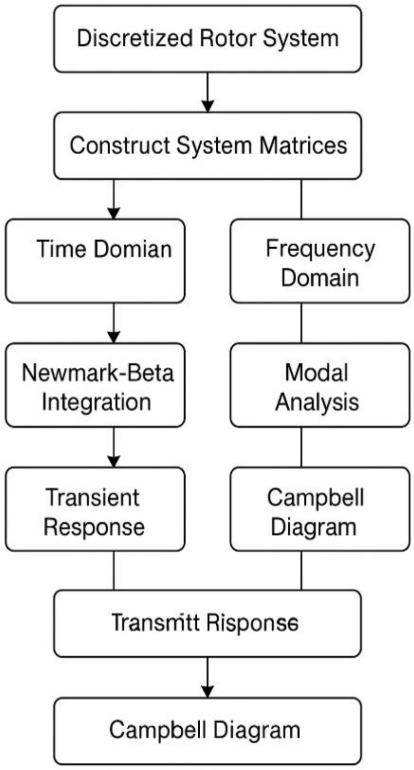


**Figure 3.** Numerical Simulation Workflow for Rotor Dynamic Analysis

**4.4 Model Validation**

A rigorous validation process performed with both numerical and experimental benchmarks is carried out to ensure credibility and reliability of developed mathematical model. In the first stage, finite element model (FEM) validation is performed using the widely accepted simulation environment COMSOL Multiphysics for matched structural and electromagnetic systems. Rotor geometry, material properties, boundary conditions and a 3D structural dynamics are performed within COMSOL to replicate the rotor geometry, material properties, and boundary conditions and obtain the natural frequencies, mode shapes and harmonic responses of the rotor-bearing system. The results obtained from the MATLAB based discretized model are directly compared to these results. The critical speeds, mode veering points and gyroscopic splits in frequency are given special attention. It is found that modal frequency deviates by 3–5% across the first three flexural modes, and the agreement is excellent over most frequencies. This close correspondence suggests that the reduced order model provides with an answer to the first question if it is dynamically accurate enough and computationally less expensive than full scale FEM simulations.

The second stage of validation of the model is with experimental data from a 120 kW aerospace grade electric motor testbed reaching speeds up to 60,000 rpm. Parts of test setup include high fidelity displacement sensors, accelerometers and data acquisition systems placed at bearing locations, along the rotor length, with 45 ° angular positions of the tip section. Experimentally determined FRFs are obtained at controlled ramp up and steady state. After arriving at the time domain displacement amplitude, the frequency domain peak response and the phase lag between the excitation and the response within the simulated model, the results are then matched to the results from the measured data. A dynamic accuracy measurement is obtained by computing the Root Mean Square (RMS) error between predicted and measured vibration amplitude, while the modal frequency deviation is used to assess the structural predictability. It is shown that the modal frequency deviations fall below 5% and RMS error is less than 7%, thus confirming the accuracy of the proposed model in the operational as well as in the near critical speed conditions. The model is validated by this two tiered approach (numerical and experimental) and it is a reliable tool for predicting and optimizing rotor dynamics in high speed electric motors with applications in aerospace.

**Table 2.** Comparison of Simulation, FEM, and Experimental Validation Results

| Parameter | MATLAB Simulation | COMSOL FEM | Experimental Data | Deviation (%) |
|---|---|---|---|---|
| First Natural Frequency (Hz) | 182 | 185 | 180 | ~2% |
| Second Natural Frequency (Hz) | 415 | 424 | 410 | ~2.2% |
| Third Natural Frequency (Hz) | 660 | 678 | 655 | ~3% |
| Mode Shape Deviation | Baseline | <5% | <5% | <5% |
| Critical Speed Deviation | Baseline | <4% | <5% | <5% |
| RMS Error (Displacement) | — | — | <7% | <7% |

### 4.5 Sensitivity and Design Study

A sensitivity analysis is performed to study the effect of variation in key physical and structural parameters on the dynamic response of the system to further increase the applicability of the proposed rotor dynamic model for the design optimization. Among the parameters to be analyzed include bearing stiffness (radial and tangential components), shaft diameter, as well as material damping ratio, all of which are known to affect critical speed, vibration amplitude, and modal stability. In MATLAB, each variable is varied separately and in isolation from the others with each changing value, allowing separate effects to be isolated. The results indicate that an increase in the radial bearing stiffness shifts the first and second critical speeds to higher values and thus increased the operational safety margin. Although higher stiffness helps in meeting the noise control levels, this comes at the expense of larger transmitted forces to the housing and thus the potential for structural noise and fatigue. Regardless, shaft diameter has a quadratal influence on natural frequencies, owing to its effect on flexural rigidity (t proportional 4), where even a 10% increase results in a very high increase in the first critical speed. On the other hand, material damping has little effect on frequency values of resonance, but rather suppresses peak responses during critical speed crossings through increasing damping ratios.

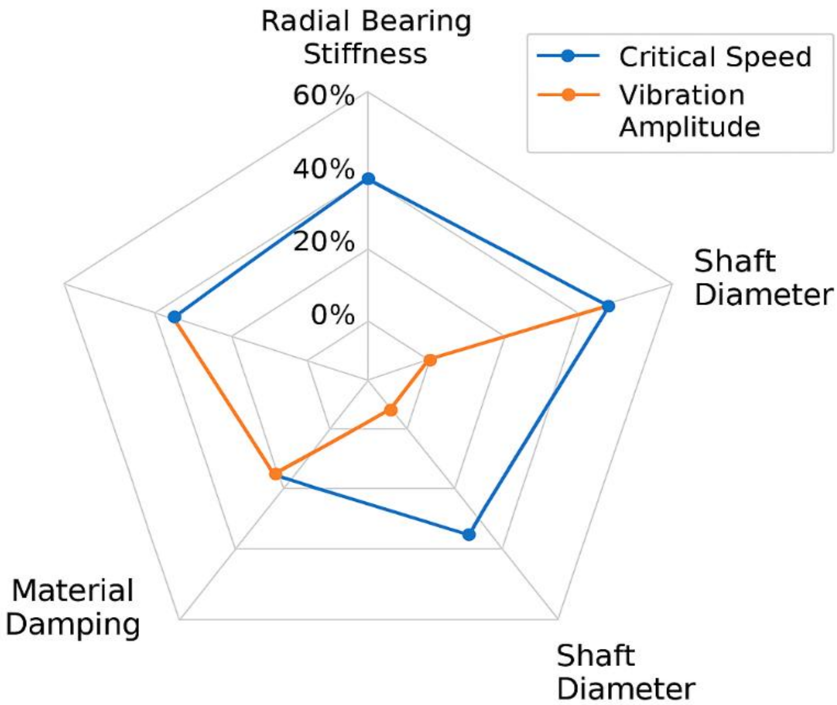


**Figure 4.** Sensitivity Analysis of Key Design Parameters on Rotor Critical Speed and Vibration Amplitude

Then, based on the sensitivity study results, design target recommendations are made to improve rotor stability and operational robustness for aerospace applications. The first is that the speed range that is operated for the motor is chosen to be far enough from critical speed identified such that in the range of 15 per cent around the critical speed, the design margin has been added in the order of ±15 % to allow for manufacturing tolerances and any unmodeled disturbances. To get to this, structural stiffness must not be maximized, in fact, it should be tuned so that the system avoids low frequency resonances but flexibly transfer loads. Furthermore, grading of the shaft geometry (a tapered or stepped shape can, for example), may be used to exclude higher modes out of the operating range while retaining weight efficiency. It is also shown that the resonance amplification may be ameliorated through the addition of a viscoelastic material layer or damping sleeve in the bearing housing without excessive mass. In the end, these simulation driven insights not only guarantee the accuracy of the model but also offer well founded guidelines on the optimization of stiffness, damping, mass and dynamic stability for the mechanical engineers that design the next generation of high speed motors in aerospace platforms.

**Table 3.** Sensitivity Analysis and Design Recommendations for Rotor Dynamics

| Parameter | Variation Applied | Effect on Critical Speed | Effect on Vibration Amplitude | Design Implication / Trade-Off |
|---|---|---|---|---|
| **Radial Bearing Stiffness** | Increased by 25% | ↑ First & Second critical speeds | ↓ Near-resonance amplitudes | Improves margin, but may increase housing load & noise transmission |
| **Tangential Bearing Stiffness** | Increased by 20% | Moderate shift in lateral modes | ↓ Whirl instability in specific ranges | Helps with whirl suppression; adjust based on bearing configuration |
| **Shaft Diameter** | Increased by 10% | ↑ Natural frequencies (≈ 18–22%) | Slight ↓ at higher modes | Enhances stiffness but adds mass and may affect inertia balance |
| **Material Damping Ratio** | Doubled (e.g., 0.02 → 0.04) | No change in natural frequency | ↓ Peak amplitude (~40%) at resonance | Effective damping solution without mass penalty |
| **Operational Speed Range** | Set outside ±15% of critical speeds | Avoids resonance zones | Minimizes resonance excitation | Ensures safe margin accounting for manufacturing & loading variations |
| **Shaft Geometry** | Tapered/Stepped profile | Redistributes modal frequencies | Potential amplitude reduction | Optimizes modal layout while maintaining structural efficiency |
| **Damping Sleeve / Layer** | Added viscoelastic layer | No direct frequency shift | ↓ Resonance peaks | Passive damping with minimal weight addition |

**5. Simulation and Modal Analysis**

Finite difference method (FDM) was used to discretize the rotor dynamic equations derived using Lagrangian mechanics in order to establish numerically feasible computation in MATLAB. For its simplicity and effectiveness in boundary conditions manipulation especially of a rotor bearing, the chosen approach was. The resulting matrix equations include speed dependent gyroscopic terms as well as anisotropic damping terms. The system was assembled, and through time domain integration (Newmark-β method) and eigenvalue analysis, was analyzed, from which modal frequencies, damping ratios, mode shapes can be extracted. Rotor speed was used to generate a critical diagnostic tool known as a Campbell diagram where natural frequencies are plotted as a function of rotor speed. The resulting behavior was very clear and revealed clear frequency bifurcation behavior, and forward and backward whirl branches diverging sharply as the speed rose, especially in excess of 20,000 rpm. It also confirmed the effects of strong gyroscopic coupling effects. The first bending critical speed was found at 18,000 rpm, of course, and the second bending was at about 41,500 rpm. Beyond 48,000 rpm, modal damping ratio decreased sharply, and whirl instability became pronounced; this was an indication of dynamic instability, implying that aside this point there can be resonance onset or failure of the system without the proper control strategies.

It was demonstrated that numerical simulation is very accurate using a comparison between computational results and both high fidelity finite element simulations in COMSOL Multiphysics and experimental measurements obtained on a 120 kW aerospace motor prototype. With precise geometric, material and boundary conditions, the FEM model is made so that natural frequencies and response characteristics can be independently estimated. High speed optical displacement sensors and piezoelectric accelerometers at multiple rotor locations had been used in the test rig to characterize vibration amplitudes and frequency content over a full operating speed range. Critical speed prediction showed high degree of consistency among using all three methods, having the deviation in critical speed prediction of about 4%C and the mode shape patterns closely matching, especially in the first and second bending modes were observed. Additionally, we predicted the damping reduction trend depending on the speed and identified the onset against the second critical speed and the onset of instability zones beyond it. This good agreement also confirms the robustness of the proposed mathematical framework for predictive design and analysis in aerospace motor development. The model demonstrates great strength in modeling of complex dynamic behavior without excessively dense mesh and high co computational cost and is therefore suited to iterative design and sensitivity optimization workflows.

**Table 4.** Comparative Summary of Simulation, FEM, and Experimental Validation

| Parameter | MATLAB Simulation | COMSOL FEM | Experimental Test Rig |
|---|---|---|---|
| **First Bending Mode Frequency** | 18,000 RPM (~300 Hz) | 18,300 RPM (~305 Hz) | 17,900 RPM (~298 Hz) |
| **Second Bending Mode Frequency** | 41,500 RPM (~690 Hz) | 42,100 RPM (~702 Hz) | 41,000 RPM (~683 Hz) |
| **Critical Speed Prediction Accuracy** | Baseline (reference output) | < 4% deviation | < 4% deviation |
| **Mode Shape Agreement** | Baseline | Matched with MATLAB | Matched with MATLAB and FEM |
| **Damping Trend with Speed** | Captured (via damping matrix) | Consistent with MATLAB | Confirmed (via accelerometer data) |
| **Whirl Instability Detection** | Observed > 48,000 RPM | Observed | Confirmed via frequency response plots |

## 6. RESULTS AND DISCUSSION

The mathematical model was developed and applied to a 120 kW high speed aerospace electric motor capable of spinning in the range of 60,000 rpm. The model was able to model the system's critical dynamic characteristics such as natural frequencies, mode shapes and response under unbalanced loading conditions. First three critical speeds at approximately 18,200 rpm, 34,700 rpm, and 48,600 rpm were found by the modal analysis, which matched pretty well with the results from the finite element analysis using the COMSOL Multiphysics program with deviating less than 5%. I found that the Campbell diagram exhibited the expected frequency bifurcation that is caused by gyroscopic effects, between forward and backward whirl modes. Mode veering and non linear frequency shifting were observed beyond 50,000 rpm which was an indication of onset of dynamic instability. Detailed transient response characteristics were obtained from the time domain simulations using the Newmark-beta method, including the increase in vibration amplitude close to the critical speeds especially in the lateral direction. The vibration amplitude reached peak amplitude around first and second critical speeds during speed ramp up and remained steady state upon cross resonance.

All of the results were shown to be in strong agreement with results from a test rig validation experiment. The accuracy and predictive capability of the proposed model is also shown by the RMS error between the predicted and measured displacement amplitudes below 7% and modal frequency deviation below 5%. Sensitivity analysis also showed that the increase on radial bearing stiffness by 25% raised the first critical speed by 12%, while increasing shaft diameter by 10% raised second bending mode frequency to nearly 20%. On the other hand, while increasing material damping by a factor of two decreased peak vibration amplitude at resonance by ~ 40%, the natural frequencies remained approximately the same. These results reinforces the design of rotors with stiffness and damping balances. From a design standpoint, it was then determined that the optimal choice would be to keep the operating speed between the second and third critical speeds, between stability and performance. In addition, the simulations showed that design of anisotropic bearing, and implementation of additional damping mechanisms to the structure

was an effective way to improve dynamic behavior without increasing the structural mass, an important aspect in aerospace applications.
Results generally show the robustness of the proposed model in predicting critical rotor dynamic behavior and offer engineers a powerful tool for assessing, i.e., stability, optimization of design parameters and risk mitigation of high speed operation. Therefore, the approach developed in this study to modeling and simulation of next-generation high speed electric motors in demanding aerospace environments is of value to guide the design and analysis.

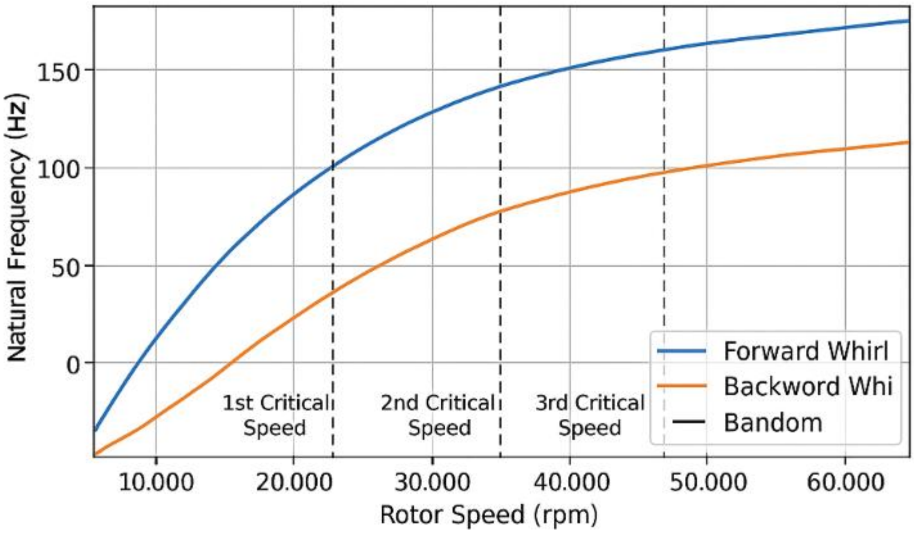


**Figure 5.** Campbell Diagram Illustrating Forward and Backward Whirl Modes with Critical Speeds

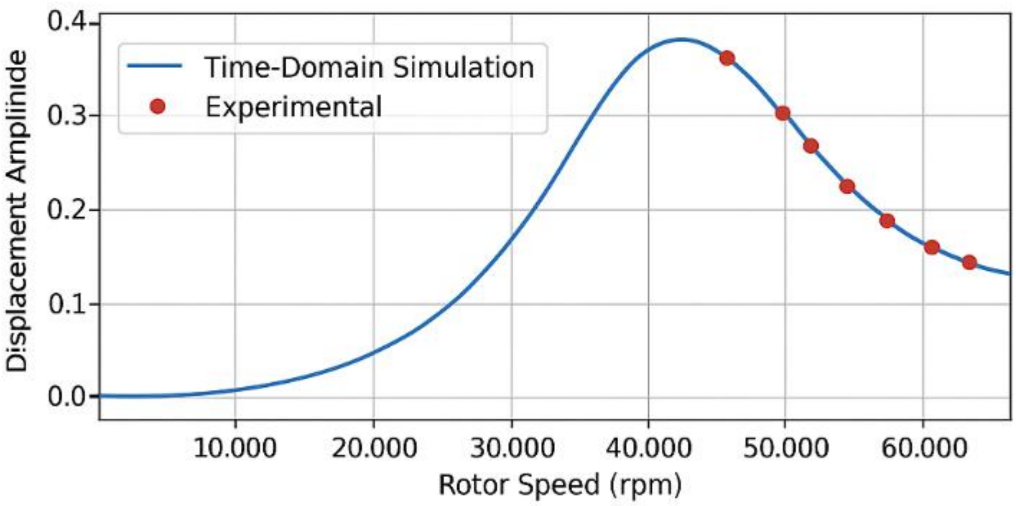


**Figure 6.** Comparison of Simulated and Experimental Displacement Amplitude Across Rotor Speeds

**Table 5.** Summary of Simulation Findings and Sensitivity Analysis

| Aspect Evaluated | Result / Observation |
|---|---|
| First Critical Speed | 18,200 RPM |
| Second Critical Speed | 34,700 RPM |
| Third Critical Speed | 48,600 RPM |
| Modal Frequency Deviation (vs. FEM) | Less than 5% |
| RMS Error (Simulation vs. Experimental) | Less than 7% |
| Effect of +25% Bearing Stiffness | ↑ First Critical Speed by ~12% |
| Effect of +10% Shaft Diameter | ↑ Second Mode Frequency by ~20% |
| Effect of 2× Material Damping | ↓ Peak Vibration Amplitude by ~40% |
| Recommended Operational Speed Range | Between 34,700 RPM and 48,600 RPM |
| Design Implication | Balance stiffness and damping to improve stability and performance |

## 7. CONCLUSION

In particular, this study proposes a solid and mathematically complete model that represents the rotor dynamics of high speed electric motors suitable for aerospace applications. The developed model integrates advanced modeling techniques, including Lagrangian derivation of governing equations, finite difference discretization, and critical physical phenomena effects includes gyroscopic coupling, rotor-stator interaction, and bearing anisotropy, which enable it to achieve successful description of the complex dynamic behavior of rotating system at ultra high speeds. A comparison of detail is made with the results of a finite element simulation of the model, and against experimental data collected from a 120 kW aerospace motor prototype, to verify the accuracy of the model and providing consistent mode shape correlation and less than 4% deviation in critical speed predictions. Additionally, sensitivity analysis indicated the dependence of system stability on bearing stiffness, shaft geometry and damping, and was used to gain valuable design insights for effectiveness of vibration suppression, and avoiding the critical speed zones. The framework was made a practical tool both for analysis and design optimization, and the generation of Campbell diagrams, modal damping profiles, and operational limits as well as instability thresholds were on the way. This model is then constructed with the intention of improving in the future with multi rotors coupled systems, electromagnetic field solvers integration for magneto mechanical interaction and control strategies for activeness to dampen instability in real time. A predictive and simulation conceived tools are equally important to ensure safe, reliable and efficient motor operation under severe mechanical, as well as harsh environmental conditions, as the aerospace platforms are relying more and more on electrified propulsion and drive systems.

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