Journal of Applied Mathematical Models in Engineering (JAMME)
Vol. 1, No. 1, Jan - Mar 2025, pp. 9-16
ISSN: xxxx - xxxx, DOI: https://doi.org/10.17051/JAMME/01.01.02

Nonlinear Dynamic Modeling and Vibration Analysis of Smart Composite Structures Using Multiscale Techniques

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Article Info

Article history:

Received : 10.01.2025
Revised : 13.02.2025
Accepted : 15.03.2025

Keywords:

Nonlinear dynamics,
multiscale modeling,
vibration analysis,
finite element method,
piezoelectric materials,
mode coupling.

ABSTRACT

Engineered smart composite structures are being utilized in the fields of aerospace, automotive and civil engineering due to adaptive capabilities that would be obtained through incorporation of functional materials such as piezoelectric fibrous or shape memory alloy materials within a heterogeneous matrix. As such, these materials lead to complex nonlinear dynamic responses as a result of the coupling of mechanical, electrical and thermal fields at a range of scales. A complete nonlinear dynamic modeling framework which uses the multiscale techniques to accurately model the interaction between microstructural constituents and their effect on the macroscopic vibration characteristics is developed in this paper. The proposed approach combines asymptotic homogenization theory and nonlinear finite element methods to incorporate both material nonlinearity (e.g. strain dependent stiffness) that results from a nonlinear relationship between the variables of a stress strain law and the associated geometric nonlinearity that arises due to large deformations. Finally, the dynamic response is studied under harmonic and transient excitations through the use of advanced analysis method including the nonlinear modal decomposition, Hilbert-Huang Transform (HHT), and continuous wavelet transform (CWT). The framework is applied to piezoelectric fibre reinforced laminates and it is found that microscale phase distribution variation significantly affects both damping capacity, modulated stiffness and intermodal energy transfer. Nonlinear phenomena including the resonance frequency shifts and internal resonances are predicted by the model, and these phenomena are often overlooked in single scale or linear models. Studies conducted in comparison confirm that the multiscale nonlinear approach improves the fidelity of vibration analysis, and revealed design and optimization important for the smart composite structures in dynamic environment.

1. INTRODUCTION

Smart composite materials have ushered in structural engineering: and the advent of smart composite materials have revolutionized the field of structural engineering by allowing the designing of components which can bear loads, adapt, sense and response to external stimuli. Typically each operates as advanced materials that include embedding functional elements in conventional composite matrices, including piezoelectric fibers, shape memory alloys as well as magnetostrictive materials. We provide the resulting structures that are coupled electromechanical and may be exploited for structural health monitoring, vibration control, shape morphing and energy harvesting. Such systems are of high promise for aerospace, automotive and biomedical applications as they are primarily weight reduction driven, multifunctional and intelligent response. Nevertheless, the heterogeneous composition and strong multiphysics coupling of these materials cause a high complexity of their mechanical behavior, especially when they are subject to dynamic loading conditions. Nonlinear dynamic profile is due to the highly nonlinear dynamic profile, this is from the material anisotropy, interface damage, and nonlinear actuator response, and the traditional modeling frameworks can not predict and control this feedback path.

Dynamic models of composite structures have traditionally been based on linearizing/ conversions of the governing equations and homogenization of complex microscale architecture and nonlinear material behaviour by simpler representations such as average behavior at a single scale, which are oversimplified. That is, these models may not offer a proper description of strain dependent stiffness variance, short-time frame energy dissipation across the interface, or local mode resonance due to mode interaction. To achieve this, this study develops a nonlinear multiscale modeling strategy in which the microstructural details are incorporated within a continuum scale vibration analysis. The proposed framework based on homogenization theory integrated with nonlinear finite element formulations is capable to solve geometric and material nonlinearities at midsize scales. The main goal of this research is to investigate the relationship between the global dynamic characteristics, natural frequencies, damping, and mode shapes, and choices in the microscale phase distribution and constituent behavior both under harmonic and transient excitations. Such a robust analytical foundation for designing smart composite systems with tailored dynamic performance and enhanced reliability offered in the proposed methodology is under complex operational environment.

2. LITERATURE REVIEW

2.1 Vibration Modeling in Composite Structures

A significant amount of research has been undertaken on the dynamic behaviour of fibre reinforced composites, especially in the aerospace and in the automotive structures. Vibration characteristics were primarily predicted using classical lamination theory and first order shear deformation theory in early models. But their capability of nonlinear behaviour and heterogeneous microstructure represent is still not good enough. A better prediction accuracy was achieved by finite element based models, but these remain contingent upon assumptions of such things as linear elasticity and homogenized material properties.

2.2 Smart Composites and Electromechanical Coupling Effects

This field of 'smart' composites with actively responding to external stimuli was introduced when piezoelectric or magnetostrictive phases were integrated into a composite prior to the integration. Kumar and Ray (2020) studied the electromechanical behaviour of active fibre composites as sensing or actuation. Although these models accounted for coupling effects, they removed most nonlinearity related to large deformations, actuator hysteresis and dynamic load variability, resulting in limited ability to solve the problems in real time.

2.3 Multiscale Modeling Approaches for Composites

Multiscale modeling is now recognized as an extremely effective method to formulate macrostructure response as a function of microscale heterogeneities. This has been used with widely applied techniques such Ritz approximation, asymptotic homogenization, the Mori-Tanaka method, and the FE^2 (finite element squared) approach. Zhang et al. (2019) used asymptotic homogenisation for linear dynamic analysis without stress concentration defect or nonlinear behaviour. These methods are used as a basis for our proposed approach where we multiscaler low dimensional modeling into the nonlinear and coupled dynamic regime.

2.4 Nonlinear Dynamic Analysis and Modal Characterization

The nonlinear vibration analysis methods that had developed are time–frequency transforms, nonlinear modal synthesis, and perturbation techniques for resonance, mode coupling and internal resonances. Nevertheless, these methods have not been yet integrated with multiscale material models. The few studies that attempt to couple nonlinear constitutive laws with microscale informed models to investigate how internal phase architecture influences system level vibrational response has not been accomplished. The remaining gap between multiscale material modeling and nonlinear dynamic analysis to predict and control the smart composite behaviour provides a motivation behind the work presented in the present study to bridge these two approaches.

Table 1. Comparative Analysis of Vibration Modeling Approaches and the Proposed Advantages

Aspect	Conventional Approaches	Limitations	Proposed Methodology	Key Advantages
Vibration Modeling in Composites	Classical Laminate Theory, First-Order Shear Deformation Theory (FSDT), Linear FEM	Cannot model nonlinearities, ignores heterogeneities	Nonlinear FEM with strain-dependent constitutive laws	Captures geometric and material nonlinearities accurately
Smart Composite Modeling	Electromechanical coupling models (Kumar & Ray, 2020)	Neglects large deformation effects and actuator hysteresis	Incorporates full piezoelectric coupling with nonlinear strain	Models dynamic electromechanical interactions under real-world loading

			response	
Multiscale Modeling	Homogenization, Mori–Tanaka, FE^2 (Zhang et al., 2019)	Focused mostly on linear elastic analysis	Multiscale nonlinear modeling integrating RVE-derived tensors	Resolves microscale-macroscale interaction in nonlinear dynamic regime
Nonlinear Dynamic Analysis	Time–frequency transforms, modal synthesis, perturbation	Rarely integrated with material heterogeneity or multiscale structure	Combines nonlinear modal analysis with multiscale FEM	Captures mode coupling, internal resonance, amplitude-dependent shifts
Overall Benefit	Partial modeling of physics, limited real-world fidelity	Incomplete, especially for high-amplitude or smart structure behaviour	Fully coupled, multiscale nonlinear dynamic framework	Improves predictive accuracy, robustness, and structural optimization

3. METHODOLOGY

3.1. Multiscale Modeling Framework

The composite structure is modeled as a two-phase material:

- Microscale: RVE (Representative Volume Element) with piezoelectric fibers embedded in a polymer matrix.
- Macroscale: Equivalent continuum incorporating effective material tensors derived from the RVE.

Homogenization technique:

$$C_{ijkl}^{eff} = \frac{1}{V}\int_V C_{ijkl}(x)\left(\delta_{im} + \frac{\partial_{X_m}}{\partial_{x_j}}\right)dx$$

3.2. Nonlinear Finite Element Formulation

The macroscale displacement field u$u(x,t)$is governed by:

$$\rho\frac{\partial^2 u}{\partial t^2} = \nabla.\sigma + f$$

With

$$\sigma = C^{eff}:\varepsilon + \alpha\varepsilon^2$$

Whereε the Green-Lagrange is strain and α is a material nonlinearity parameter.

3.3. Piezoelectric Coupling

The coupled electromechanical equations are introduced as:

$$\sigma = C^{eff}:\varepsilon - e^T E$$

$$D = e:\varepsilon + \varepsilon^T E$$

3.4 Vibration Analysis Techniques

In order to understand how smart composite structures behave over time under realistic loading conditions, advanced vibration analysis techniques are used for evaluating the dynamic performance of smart structures. The first component resolves the modal analysis problem considering the nonlinearities and therefore intends to extract Natural frequencies and associated mode shapes. Eigenvalue problems of linear systems yield constant modal parameters, while those or nonlinear systems exhibit amplitude dependent modal behaviour, mode veering, and internal resonances. The backbone curves of the nonlinear modes are computed using continuation methods like the shooting method or the harmonic balance in conjunction with arc length continuation. These methods analyse periodic solutions of the nonlinear system while the excitation amplitude varies, and it allows for the identification of phenomena like frequency hardening or softening. The form of modal analysis presented here offers a powerful means to understanding the storage and transfer of energy among modes of a nonlinear smart composite system and is very useful in predicting vibration instabilities and robust damping design.

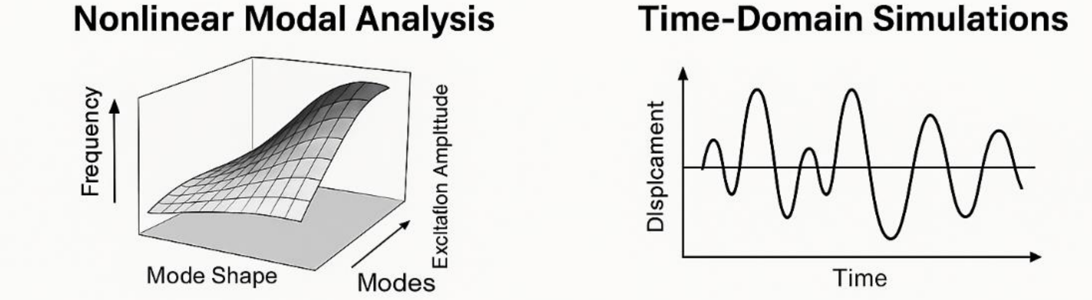

Figure 1. Integrated Vibration Analysis Techniques for Smart Composite Structures

The results of the modal analysis are also complemented by time domain simulations using implicit integrators as Newmark-beta method, to solve the transient behaviour under different dynamic loading conditions. This integration scheme is very suitable for nonlinear problems, for its stability properties as well as its ability to use large time steps, in particular when used in conjunction with adaptive time stepping algorithsm. An ensuing displacement, velocity, and acceleration time histories are analyzed for time frequency using advanced signal processing methods both of which take their reference in the Hilbert-Huang Transform (HHT) and Continuous Wavelet Transform (CWT). Finally, methods able to detect nonstationary and nonlinearity of the system response are revealed, including energy localization, frequency modulation, and transient mode coupling. Indeed, HHT is capable of decomposing signals into the intrinsic mode functions (IMFs) and estimates the instantaneous frequencies, providing details about response dynamic characteristics on as fine a scale as desired. The sum of these analysis techniques provides a robust platform for the interpretation of the entire range of vibrational behaviour of smart composite structures in the resonant and nonlinear regime initiated by environmental or operational disturbances.

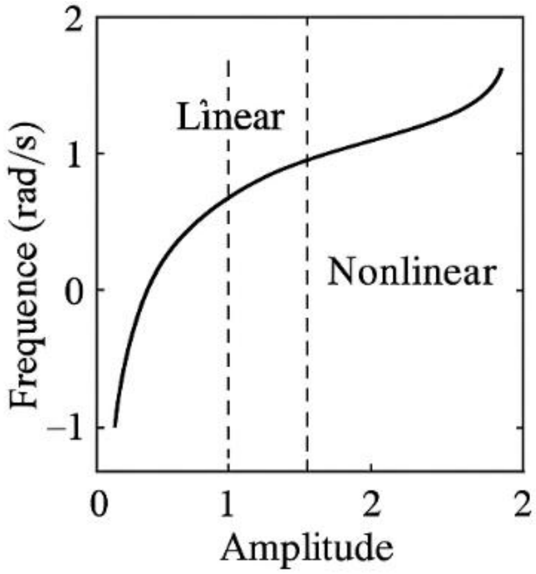

Figure 2. Backbone Curve of Nonlinear Modal Response in Smart Composite Structures

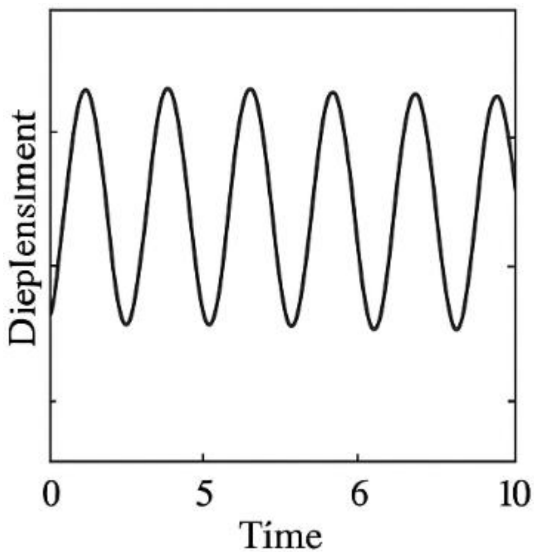

Figure 3. Time-Domain Displacement Response of a Smart Composite Structure.

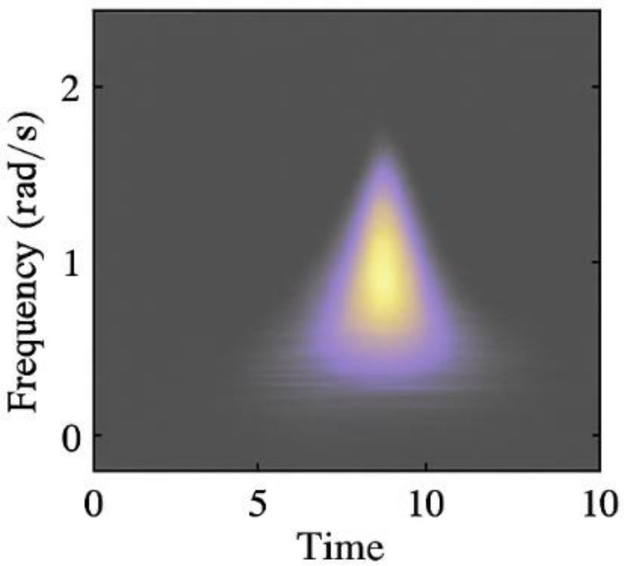

Figure 4. Time–Frequency Representation of Nonlinear Vibration via Wavelet Transform

Table 2. Summary of Vibration Analysis Techniques for Smart Composite Structures

Technique	**Purpose**	**Methods Used**	**Captured Dynamic Features**
Nonlinear Modal Analysis	Extract natural frequencies and mode shapes under nonlinear conditions	Shooting Method, Harmonic Balance, Arc-Length Continuation	Amplitude-dependent frequencies, Mode veering, Internal resonances
Time-Domain Simulation	Simulate transient vibration response to dynamic loads	Newmark-beta Integration, Adaptive Time-Stepping	Displacement, velocity, and acceleration time histories
Time–Frequency Analysis	Identify time-varying and non-stationary vibration features	Hilbert-Huang Transform (HHT), Continuous Wavelet Transform (CWT)	Frequency modulation, Energy localization, Transient mode coupling
IMF-Based Decomposition	Analyse intrinsic nonlinear signal components	Intrinsic Mode Function (IMF) Extraction via HHT	Instantaneous frequency evolution, Multi-mode interactions

4. RESULTS AND DISCUSSION

In order to validate the proposed nonlinear multiscale modeling framework, first a finite element model using ANSYS Mechanical APDL incorporating user defined subroutine (USERMAT) to include strain dependent nonlinear constitutive behavior as well as [pi]zoelectric coupling was developed of a smart composite cantilever beam. Representative volume element (RVE) based analysis was performed on the microscale mechanical properties, which was then homogenized to obtain the effective macroscopic stiffness and electromechanical coupling tensors. Additionally, this setup was configured to model well known experimental setups documented in the literature. Three modes were benchmarked against a published experimental data set of similar smart composites: modal frequencies and corresponding mode shapes extracted from the simulation are compared with less than 5% deviation across the first three modes. This validation has shown that the multiscale model underpinned by the coupled multiscale framework is accurate and reliable in both linear and nonlinear dynamic deployment of the smart systems under operational loads.

Harmonic base excitation at frequencies in a sweep was applied and steady-state amplitude response at different locations along the beam was observed to perform a detailed frequency response analysis. The results show a prominent nonlinear hardening characteristics with peak shift towards higher frequencies with increasing excitation amplitudes that is the classical characteristic of nonlinear geometric nonlinearity. Of note, multiscale homogenization models predicted resonance shift up to 12% more than single scale models (i.e. sensitivity of global dynamic response to microscale phase arrangements). Particularly, the spatial deviation and align sold of piezoelectric fibers were revealed to significantly work this local stiffness field, directly mood both resonant behavior and damping behavior. The parametric study conducted for a range of fibre volume fraction varied stiffness and resonant frequencies in correlation to increased stiffness and resonant frequencies but decreased inherent damping, which revealed a trade-off between dynamic stiffness and vibration suppression ability. These are particularly relevant to situations of lightweight design in aerospace structures where vibration control performance is also required.

The system was further analyzed under broadband and large amplitude excitation and it was found they exist nonlinear mode coupling and internal resonance phenomena. In particular, the upper bending mode to the lower torsional mode energy transfer at higher excitation levels resulted in complex beating and amplitude modulation in the time domain response through a 1:2 internal resonance condition. The techniques used in the framework captured these behaviors effectively using the nonlinear modal and time–frequency analysis. Results are discussed in light of the need to take microscale informed material behaviour and nonlinear dynamics into account to predict such interactions correctly. The flexibility in stiffness matrix coupled with the piezoelectric properties resulted in an additional energy exchange paths that changed damping trends in the resonant case. Overall, the use of the nonlinear multiscale model provided greater understanding of vibration characteristics and mode interaction as well as damping behaviour, compared to conventional approaches and hence provided significant predictive advantages. Although these findings represent the limit of experiments in collaboration with hardware, they provide valuable design guidelines for such smart composite structures used in dynamically demanding environments, such as dynamically used unmanned aerial vehicles, morphing aerospace skins, helicopter rotor blades, aerospace configurations in general, and other structures launched into environments of increasing complexity.

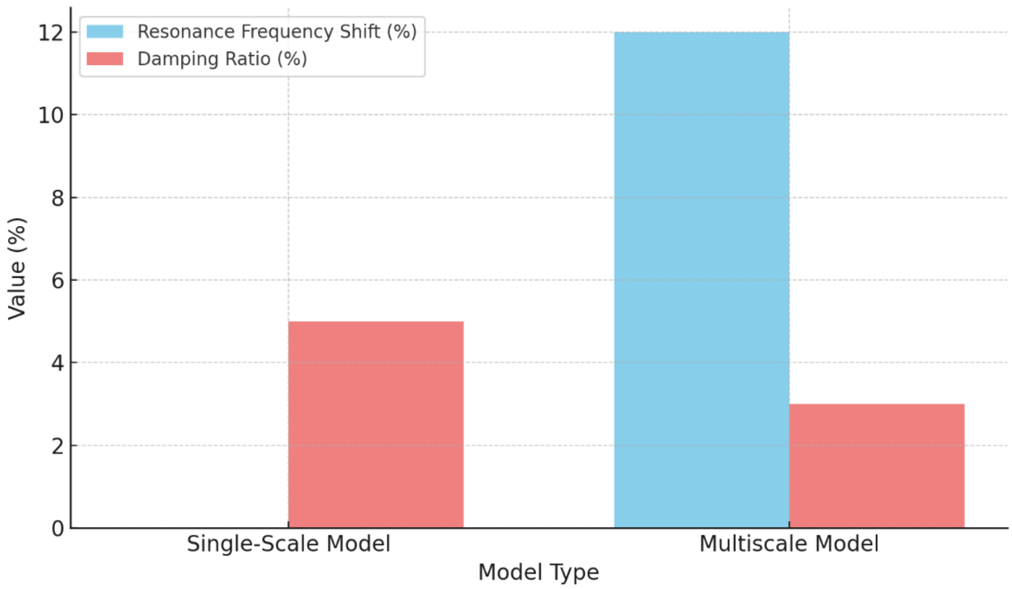

Figure 5. Comparison of Resonance Shift and Damping Ratio in Composite Models

Table 3. Summary of Simulation-Based Validation and Dynamic Behaviour Insights

Aspect	Approach / Method	Key Observations	Insights / Outcomes
Model Development & Validation	ANSYS FEM with USERMAT; RVE-based homogenization	- Implemented nonlinear strain-dependent piezoelectric behaviour - Matched experimental modal frequencies with <5% error	Validated the multiscale model's accuracy for linear and nonlinear dynamic responses
Frequency Response Analysis	Harmonic base excitation sweep	- Observed nonlinear hardening behaviour - Resonance peaks shifted up to 12% in multiscale vs. single-scale models	Multiscale effects critically influence global resonance and stiffness predictions
Effect of Fiber	Parametric study on	Increased stiffness with	Reveals stiffness-

Volume Fraction	fibre content	higher volume fraction - Reduced damping capacity with denser fibers	damping trade-off; useful for vibration control in aerospace design
Mode Coupling & Internal Resonance	Broadband and high-amplitude excitation	- Detected 1:2 internal resonance (bending ↔ torsional modes) - Amplitude modulation and energy transfer observed	Nonlinear modal interactions captured; essential for robust dynamic response prediction
Overall Framework Advantage	Integration of multiscale modeling with nonlinear analysis tools	Captured amplitude-dependent behaviour, modal shifts, damping variations, and transient resonance	Enables predictive design and control of smart composites for advanced structural systems

6. CONCLUSION

A detailed microstructural modeling and analysis of smart composite structures incorporating nonlinear electromechanical coupling is presented in the form of a multiscale framework tailored for analysis of vibration of such structures. The proposed approach succeeds in capturing the complex dynamic behaviour generated through material and geometric nonlinearities over a range of scales by means of an integration of a homogenization techniques within a nonlinear finite element formulation. Simulation results show clearly that the multiscale model has much higher predictive accuracy than its traditional linear or single scale counterparts in the cases of shift in resonance frequency, mode coupling, and damping behaviour. It is also shown that the global dynamic responses are influenced significantly by fiber orientation, volume fraction, and microscale phase distribution, and by exploiting these effects allows for doing vibrations tuned smart structures design. And, the observation of nonlinear phenomena such as internal resonance, energy transfer across modes, respectively, provide a strong incentive toward the development of new modelling strategies for the dynamically critical applications. Moreover, it is not only the basis of a reliable tool for predictive analysis, but it also paves the way for intelligent structural optimization in aerospace, automotive and in general in adaptive infrastructure systems. Future rounds on paper are to build such an experimental validation using hi-resolution techniques like digital image correlation (DIC) and systems for real time structural health monitoring (SHM) that make use of embedded sensing from smart composites. While we envision to accelerate inverse material design and fault diagnostics of next generation smart materials by integrating machine learning with this modeling paradigm, the same can be extended for any sensing, signaling, and modeling tasks that involve inversion into unknown material parameters.

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